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Modelling the Propagation of a Directional Wave Spectrum in the Marginal Ice Zone

Fabien Montiel¹, Vernon A. Squire¹ & Luke G. Bennetts²

*¹Department of Mathematics and Statistics, University of Otago, PO Box 56
Dunedin 9054, New Zealand*

*²School of Mathematical Sciences, University of Adelaide, Adelaide, South Australia 5005, Australia
E-mail: fmontiel@maths.otago.ac.nz, vernon.squire@otago.ac.nz, luke.bennetts@adelaide.edu.au*

The single ice edge band model of Montiel et al. (2014) is extended to depict the progression of a prescribed directional wave spectrum as it enters and traverses a marginal ice zone (MIZ) composed of ice floes of random sizes and locations present at some concentration. This is achieved by considering the MIZ to be assembled from many contiguous parallel ice bands. Although the wave energy entering each constituent band is multiply-scattered by the elemental ice floes that make up the band, the complicated interactions that arise can be consolidated into continuous sums of plane waves travelling at all possible angles which constitute the reflected and transmitted directional spectra. The latter sum passes into the next band in sequence, which allows the entire MIZ to be modelled expediently. Comparisons with field data (Wadhams et al., 1986) indicate good qualitative agreement for single bands of ice, with improved reconciliation as ice concentration is increased towards the usually high concentration values observed within ice edge bands in Nature. As directional seas travel through MIZs, they are found theoretically to broaden in their directional spread with the degree of broadening increasing with penetration; short waves quickly become directionally isotropic, while longer waves remain collimated for greater distances into the pack ice. This is compatible with Wadhams et al. (1986). The waves are preferentially attenuated en route, as well, again with the short period end of the spectrum being affected first. The overarching motivation of this work is the optimum assimilation of wave-ice interactions into ice/ocean models and oceanic general circulation models.

1. Introduction

Assisted by so-called Arctic amplification (Jeffries et al., 2013), the onset and acceleration of global climate warming has triggered a regime shift in the ice-covered Arctic region during the last two decades which has resulted in an alarming retreat in the sea ice cover — especially during the summer months. The dynamics of the atmosphere/ice/ocean coupled system are responding to these changes via an accentuation of extreme weather events and an attendant amplification of ocean wave power (Young et al., 2011), contributing to the destruction of the sea ice canopy. A mixed region of open water and broken ice floes, referred to as the marginal ice zone (MIZ), now typically extends over tens of kilometres along the periphery of the Arctic ice cover during the summer melt season. This enhances additional ice seasonal melt and exposes the interior pack ice to rougher seas and consequential breakage.

Current Arctic ice/ocean models and oceanic general circulation models do not account for ocean wave-sea ice interactions, even though the ocean wave spectrum is the dominant factor controlling floe size distribution (FSD) in the MIZ. Moreover, it has recently been suggested that waves play a significant role in governing regional sea ice extent in the Southern Ocean (Kohout et al., in press). Our goal, therefore, is to improve the forecasting capabilities of such models, which have been unable thus far to predict the observed recent rapid decline in Arctic sea ice accurately (Jeffries et al., 2013).

A two-dimensional (2D) model has recently been proposed to assimilate ocean wave attenuation and ice floe breaking in the MIZ in a coupled Arctic ice/ocean model (Squire et al., 2013; Williams et al., 2013a,b). The physics involved in the wave attenuation phenomenon is primarily based upon energy conservative wave scattering. The model ignores multi-directional scattering effects, however, as well as the directionality of the ocean wave spectrum, which have been found to be significant near the ice edge (Wadhams et al., 1986).

To remedy this shortcoming, we have devised a new 3D model that accounts for directional scattering of realistic directional wave spectra in a MIZ with a randomized floe size distribution (FSD). The MIZ is composed of an arbitrary finite distribution of circular ice floes that are clustered into multiple adjacent bands of arbitrary width. This extends the multiple row approach considered by a number of authors (see, e.g., Bennetts and Squire, 2009; Peter and Meylan, 2009; Bennetts et al., 2010), where each row has infinite extent and a regular periodic distribution of floes. Analogous to this work, we recently proposed a solution to the single band problem and analyzed the effect of randomizing the FSD on the response of the system (Montiel et al., 2014). We were able to reproduce qualitatively the wave attenuation and widening of the directional spread experienced by a wave spectrum as it passes through and interacts with an ice edge band. Preliminary results for the multiple band problem were also presented at the American Geophysical Union Fall Meeting 2013 in San Francisco (see Montiel et al., 2013).

In the current paper, we first extend the single band analysis of Montiel et al. (2014), showing additional results for a realistic band parametrized using the observations of Wadhams et al. (1986). These new results are based on a new technique to generate more realistic FSDs, so we will evaluate how it affects wave attenuation and directional spreading. We then discuss the evolution of a directional wave spectrum in a MIZ of dimension 15×15 km, composed of over 2500 floes. Preliminary results for wave attenuation and directional spreading are analyzed over a range of wave periods. This demonstrates the capabilities of our model and its potential to generate more realistic estimates of wave attenuation in the ice-covered Arctic Ocean for assimilation in ice/ocean models.

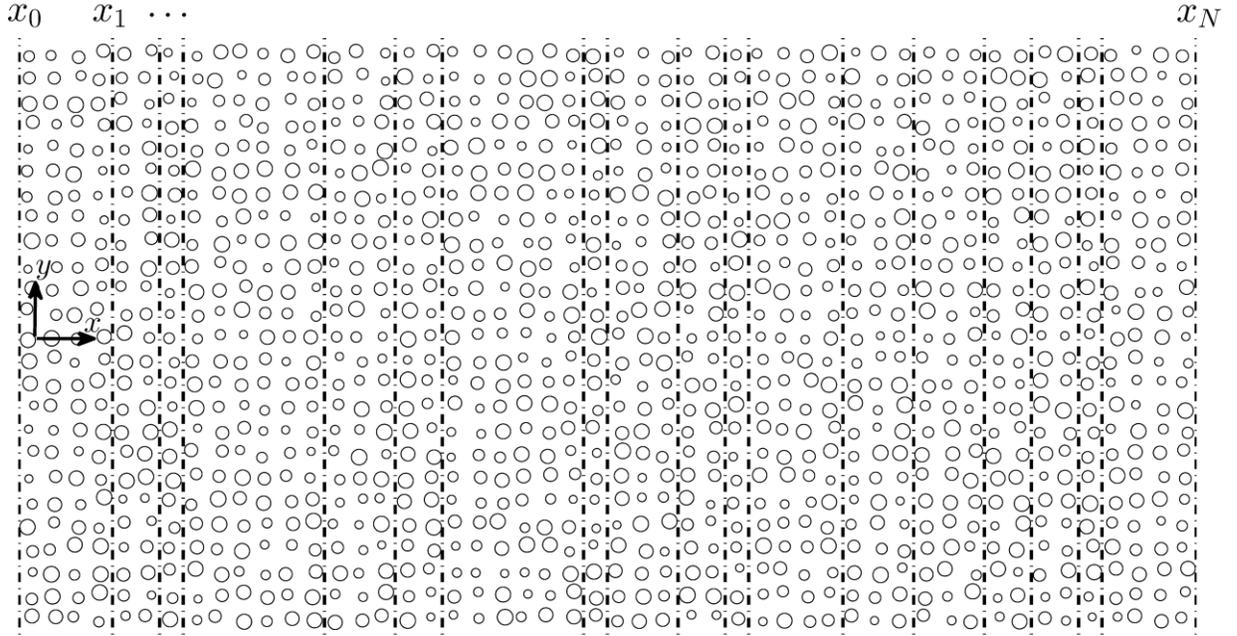


Figure 1. Random distribution of 1250 circular ice floes clustered into N bands.

2. Methods

We construct a MIZ by subdividing the ocean surface into N adjacent infinite bands of finite arbitrary width defined by $x_j < x < x_{j+1}$ ($j = 1, \dots, N$). Each band contains a finite random array of circular ice floes. Methods to generate numerically large FSDs will be discussed later. Figure 1 illustrates the geometry of the problem for a MIZ containing 1250 floes.

We input a single frequency directional wave spectrum evolving in the positive x -direction, with a standard cosine-squared angular spreading function of the type

$$S(\chi) = (2/\pi) \cos^2(\chi), \quad -\pi/2 \leq \chi \leq \pi/2, \quad [1]$$

noting that the spreading function may be defined arbitrarily. The governing equations of the hydroelastic problem can be found in Montiel et al. (2014). They arise from linear water wave theory applied at finite constant depth, to describe the motion of the fluid, and thin-elastic plate theory to reproduce the flexural motion of the ice floes. We also make the assumption of time harmonic motion with period T . We have assembled a solution method that allows us to characterize the wave field at each band interface $x = x_j$.

The solution to the single band problem is detailed in Montiel et al. (2014), so only a brief outline of the method is given here. Under an incident wave forcing characterized by the directional spectrum [1], scattering processes produce reflected and transmitted wave components that are represented as a continuous sum of plane waves travelling at all possible angles. As a result, we seek mappings between incident and reflected/transmitted spectra to describe the response of the system. This is achieved in two steps as follows:

1. derive the solution to the multiple scattering problems between all the floes (in local polar coordinates associated with each floe) using the interaction theory (see Kagemoto and Yue, 1986; Peter and Meylan, 2004);
2. apply a polar to Cartesian coordinate transform for the outgoing circular waveforms arising from each ice floe.

Expressing the solution of the multiple scattering problem in Cartesian coordinates gives the multi-directional plane wave representations of the reflected and transmitted wave fields. A similar method yields the reflected and transmitted fields for an incident wave field travelling in the negative x direction. Combining the two problems and discretizing the angular range, we obtain a solution in the form of a scattering matrix, which maps the incident wave amplitudes to the scattered wave amplitudes. Extending the solution method for multiple bands is then straightforward, using well established methods for multiple scattering in one dimension (see, e.g., Bennetts, 2011).

3. Results and Discussions

We show results for a single band first, extending the analysis conducted in Montiel et al. (2014). We parameterize the band using data by Wadhams et al. (1986) collected as part of the Marginal Ice Zone Experiment 1984 campaign (MIZEX-84) in the Greenland Sea. We define a band of sea ice with width 210 m and length 525 m. In the paper by Wadhams and colleagues, the FSD is provided after binning the floe properties into 5 categories for computational purposes. The bins are defined by their characteristic radii 6.25, 12.5, 17.5, 27.5 and 50 m, and proportions of the total ice-covered surface area 20, 30, 30, 10 and 10%, respectively. The ice concentration was not provided by the authors although we expect it to be high, as explained in Montiel et al. (2014). We also set the thickness to 2 m for all floes and the following typical material properties: density 922.5 kg/m³, Young's modulus 6 GPa and Poisson's ratio 0.3. The effect of these properties on the subsequent results has not been investigated in this paper, although we do not expect them to have a strong influence on the scattering properties of this three-dimensional model.

Two methods are used to generate the FSD in our model:

1. divide up the band using a rectangular grid of 6 by 15 square cells and randomly position a single floe in each cell. To limit the size of the cells taken, we ignore the influence of larger floes, i.e. with 25.5 and 50 m radius which account for only 1.5% and 0.4% of the total number of floes. By this means, we obtain an average ice concentration of 35%;
2. use a circle packing algorithm to position floes iteratively at random in the band, making sure floe boundaries never intersect. Floes are added to the domain until none of the sizes considered can fit. This time, we ignore the contribution from the biggest floes (50 m radius) and the smallest floes (6.25 m radius), arguing that they would have little effect on wave scattering. This method produces an average ice concentration of 68%.

We have run tests using both methods for periods in the range $T = 3-10$ s. Following Wadhams et al. (1986), we consider the leeward to windward energy density ratio, defined as the transmitted to incident-plus-reflected energy ratio in our context. We compute this quantity using an ensemble average over 50 random realizations of the band. The responses are displayed in figure 2 for both FSD generation methods. The observed data of Wadhams et al. (1986) have also been reproduced, although we must acknowledge that much uncertainty is associated with that process.

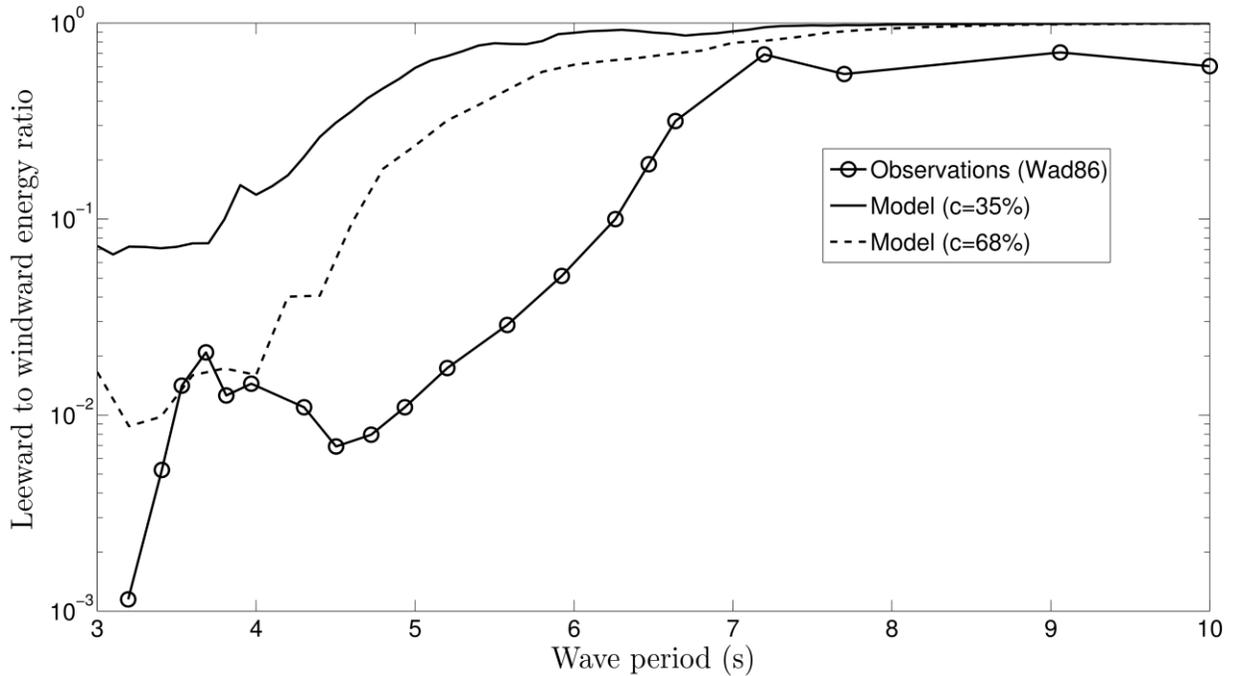


Figure 2. Observed and simulated leeward to windward energy density ratio against wave period.

It is seen that the circle packing method lowers the discrepancy between simulated and observed data appreciably, suggesting that ice concentration is critically important to wave attenuation. The agreement is good at low periods and reasonable at high periods, but a good deal of discrepancy is evident in the middle range. Many factors could explain part of this discrepancy, such as the uncertainty in the incoming wave spectrum, the measurement errors (no error bars are given in the original data), the uncertainties in geometry (FSD, band extent and width), and sources of dissipative processes that are not included in our energy-conserving model.

We now consider a MIZ made of 50 bands of width 300 m, each composed of 51 floes (i.e. 2550 floes). In each band, the FSD is obtained by random perturbation of the regular arrangement with constant centre-to-centre spacing 300 m, which defines the mean characteristics of the FSD. We randomly define thicknesses in the range 1–2 m, radii of 50–100 m and position in 100 m intervals around the mean centre location in both x and y directions. The mean ice concentration is then 20%. All subsequent results are averaged over 30 realizations of the MIZ.

We define $E_j^l(\chi)$ and $E_j^r(\chi)$ as the left-travelling and right-travelling wave energy angular spectra at $x = x_j$, respectively. Figure 3 shows the evolution of the energy spectra through the MIZ for two wave periods, 6 and 10 s. For both periods, the wave energy decreases with distance from the left ice edge, although the attenuation is more important for shorter waves, as expected. We also observe a widening of the angular spread with distance from the ice edge. In particular, the right-travelling component of the wave energy at $T = 6$ s becomes isotropic when it exits the MIZ at $x = x_{50}$, a phenomenon that has been observed commonly for short waves in the real MIZ.

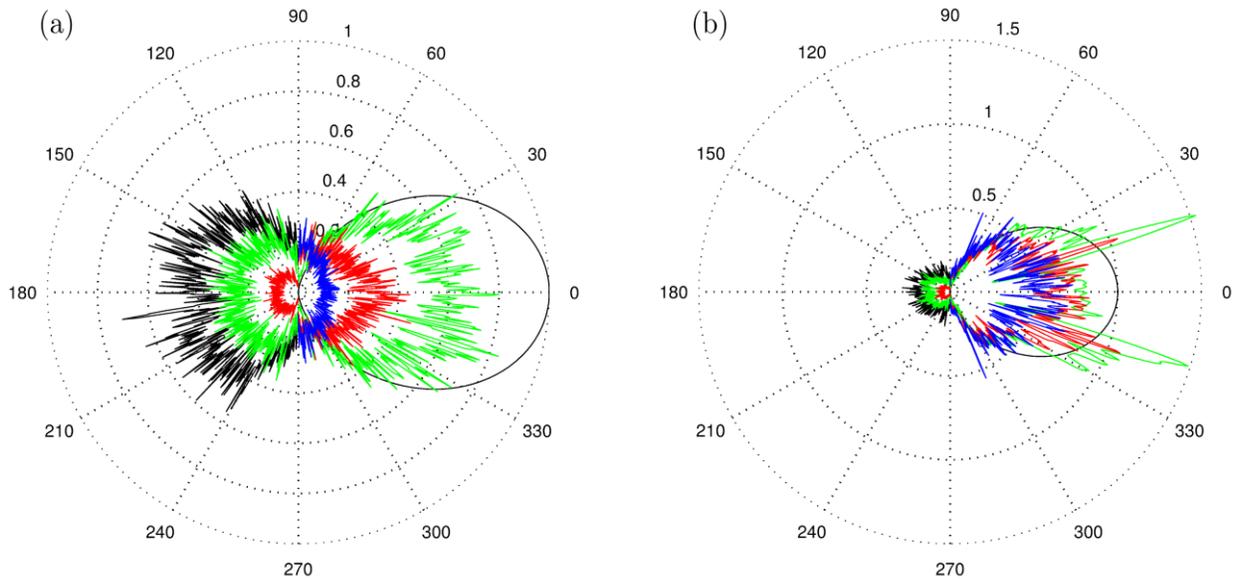


Figure 3. Polar plot of wave energy at $x = x_0$ (black), $x = x_{10}$ (green), $x = x_{30}$ (red) and $x = x_{50}$ (blue), for wave periods (a) $T = 6$ s and (b) $T = 10$ s.

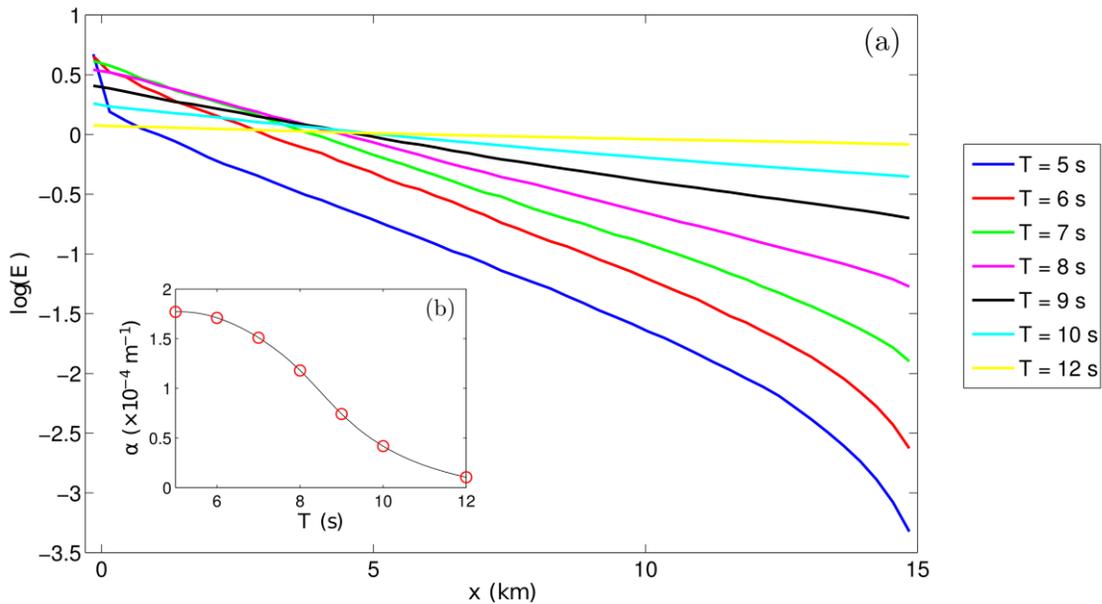


Figure 4. Natural logarithm of the wave energy through the MIZ for wave periods $T = 5$ – 12 s. The attenuation coefficient is then plotted against period in sub-plot (b).

Following well established observations of wave attenuation in the MIZ (e.g. Squire and Moore, 1980), we assume that the energy decays exponentially in the form $E = E_0 \exp(-\alpha(x - x_0))$. The energy is calculated at each strip boundary as the sum over all angles of the directional energy spectrum, relative to the incident spectrum. Figure 4a shows the natural logarithm of the energy through the MIZ. We observe a linear portion for all periods, which we use to obtain the

attenuation coefficient α from a standard least-square interpolation. The attenuation coefficient decreases as the period increases (see figure 4b), in agreement with past experimental and theoretical studies. We note that near the end of the MIZ, wave energy attenuation increases. This is explained by the fact the MIZ has finite extent and the presence of the open ocean after the MIZ induces an absence of backscattered waves which augments the apparent attenuation.

4. Conclusions

In this paper, we extended the single band model discussed in Montiel et al. (2014) to describe the evolution of a directional wave spectrum in a MIZ with random floe size distribution. Our intent was to bridge the gap between the single band model and the multiple band model, which then allows us to consider ocean wave propagation through a MIZ of large extent (15×15 km); similar in size to standard grid size in ice/ocean models incidentally. We can summarize our main findings as follows:

1. model/observation comparisons of wave transmission for a single band, as conducted by Montiel et al. (2014), show an improved agreement when the FSD is generated with a higher ice concentration as is normally observed for bands;
2. widening of the directional wave spectrum with distance from the ice edge is predicted by our model for a large MIZ, in accord with field observations. The effect is particularly enhanced for shorter waves;
3. exponential attenuation of wave energy is also predicted as the directional spectrum travels through the MIZ.

These preliminary results suggest that our model is appropriate to describe conservative ocean wave-sea ice interaction phenomena. More work is needed, however, to validate the model properly and to use its predicting capabilities to integrate more realistic MIZ wave processes into ice/ocean models.

Acknowledgments

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