

The solutions provided here are not always the only way to get to the correct answer, nor even the quickest.

In physics, the half life of a decaying substance is the time it takes to reduce to half its initial quantity. A certain radioactive substance, Zn-71, has a half-life of 2.4 minutes.

(1) How much of a 100 g initial sample of Zn-71 would be left after 7.2 minutes?

- (A) 25 g (B) 12.5 g (C) 10 g (D) 6.25 g

■ **(B):** 7.2 minutes is three half lives of Zn-71. This means we divide 100 by 2^3 to get 12.5 g.

(2) How long would it take for a 48 g sample of Zn-71 to decay to 3 g?

- (A) 4.8 minutes (B) 7.2 minutes (C) 9.6 minutes (D) 12 minutes

■ **(C):** After 2.4 minutes the sample would now weigh 24 g. After 4.8 minutes it would weigh 12 g, after 7.2 minutes 6 g, and after 9.6 minutes 3 g.

(3) An unknown quantity of the Zn-71 is placed in a box for 12 minutes. Afterwards, the remaining quantity of Zn-71 is found to weigh 5 g. How much Zn-71 was there when it was initially placed in the box?

- (A) 100 g (B) 160 g (C) 320 g (D) 400 g

■ **(B):** 12 minutes is five half lives of Zn-71. Multiply 5 by 2^5 to get 160 g.

Jed is running on a treadmill. He starts by walking at 6 km/h. After 20 minutes, he increases his speed to 12 km/h and runs for another 40 minutes.

(4) What distance did Jed travel on the treadmill in total?

- (A) 10 km (B) 12 km (C) 12.8 km (D) 14 km

■ **(A):** If Jed walked for an hour, he would walk 6 kilometres. He walks for one third of that time, so walks a distance of 2 kilometres. Similarly, he runs $\frac{2}{3} \times 12 = 8$ km. In total Jed travels $2 + 8 = 10$ km on the treadmill.

(5) What was Jed's average speed in kilometres per hour (km/h)?

- (A) 8 km/h (B) 9 km/h (C) 10 km/h (D) 11 km/h

■ **(C):** Jed only uses the treadmill for 1 hour.

Daniel is also running on a treadmill. He runs at a constant speed of 8 km/h, then slows down to a speed of 4 km/h. He runs for a total of 36 minutes, and completes a total distance of 4 km.

(6) For how long did Daniel run at 8 km/h?

- (A) 12 minutes (B) 18 minutes (C) 24 minutes (D) 30 minutes

■ **(C):** If Daniel runs at 8 km/h for 24 minutes, he covers a distance of 3.2 km. He then must run at a speed of 4 km/h for 12 minutes, a distance of 0.8 km, for a sum distance of 4 km.

(7) Daniel wants to run 2026 kilometres in 2026. He goes to the gym four times a week, and runs the same number of kilometres each time. How many kilometres should he run each time he goes to the gym if he wants to achieve his goal? Round your answer to one decimal place.

- (A) 6.2 km (B) 7.3 km (C) 8.4 km (D) 9.7 km

■ **(D):** Daniel will go to the gym $52 \times 4 = 208$ times in 2026. If he runs the same number of kilometres each time, the average time he runs will be the same as the amount he runs each time. Thus (rounded to one decimal place) he runs $\frac{2026}{208} = 9.7$ km per gym visit.

A cylindrical water tank is leaking water at a constant rate. In two minutes, the height of the water in the tank drops by 15 cm. In the same amount of time, the water that escaped exactly fills a 50 L container. (1 L = 1000 cm³.)

(8) What is the rate of decrease of the height of the water in meters per second? Give your answer to the nearest 2 significant figures.

- (A) 0.15 m/s (B) 0.0125 m/s (C) 0.013 m/s (D) 0.0013 m/s

■ (D): Since $15 \text{ cm} = 0.15 \text{ m}$ and 2 minutes is 120 seconds, the rate of decrease is $\frac{0.15}{120} = 0.00125 \text{ m/s}$ which to 2 significant figures is 0.0013 m/s .

(9) What is the rate of flow of the water out of the tank in cm^3/s ? Give your answer to the nearest 2 significant figures.

- (A) $430 \text{ cm}^3/\text{s}$ (B) $420 \text{ cm}^3/\text{s}$ (C) $416 \text{ cm}^3/\text{s}$ (D) $410 \text{ cm}^3/\text{s}$

■ (B): Since $50 \text{ L} = 50000 \text{ cm}^3$, the rate of flow of the water out of the tank is $\frac{50000}{120} = 416.67 \text{ cm}^3/\text{s}$ to 2 decimal places. To 2 significant figures this value is $420 \text{ cm}^3/\text{s}$.

(10) What is the radius of the tank in cm ? Give your answer to the nearest 2 significant figures.

- (A) 33 cm (B) 32 cm (C) 28 cm (D) 25 cm

■ (A): The volume of a cylinder is found using the formula $\pi r^2 h$. We have $15 \times \pi r^2 = 50000$. Thus $r^2 = 1061.03$ to 2 decimal places, and so $r = 32.57 \text{ cm}$ to 2 decimal places. To 2 significant figures this value is 33 cm.

(11) The tank is 120 cm tall. What is the total capacity of the tank?

- (A) 150 L (B) 200 L (C) 300 L (D) 400 L

■ (D): Since $15 \times 8 = 120$, the capacity is $8 \times 50 = 400 \text{ L}$.

(12) What is the probability that the sum of two six-sided dice rolls is exactly 4?

- (A) $1/12$ (B) $1/9$ (C) $1/6$ (D) $1/3$

■ (A): There are 36 possible dice rolls. Of these 3 sum to 4: $1 + 3$, $3 + 1$, and $2 + 2$. Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

(13) What is the probability that after three six-sided dice rolls neither a 2 nor a 6 is rolled?

- (A) $64/216$ (B) $125/216$ (C) $152/216$ (D) $91/216$

■ (A): The probability of not rolling a 2 or a 6 on a single die roll is $\frac{4}{6} = \frac{2}{3}$. This means the probability of not rolling a 2 or a 6 after three dice rolls is $(\frac{2}{3})^3 = \frac{8}{27}$. Since $27 \times 8 = 216$ this is equivalent to $\frac{64}{216}$.

(14) Three consecutive one digit integers are a perfect square, a perfect cube and a prime (not necessarily in that order). What is the product of these three integers?

- (A) 60 (B) 210 (C) 336 (D) 504

■ (D): The only perfect cubes with one digit are 1 and 8. Since neither 2 nor 3 are perfect squares, it follows that the perfect cube must be 8. This works, as 9 is a perfect square and 7 is prime. Take the product of the three: $7 \times 8 \times 9 = 504$.

(15) What is $\frac{2^8}{8^2}$ equal to?

- (A) 2 (B) 4 (C) 16 (D) 64

■ (B): Since $8 = 2^3$, we have $\frac{2^8}{(2^3)^2} = \frac{2^8}{2^6}$. Simplifying we get 2^2 or 4.

(16) What is half of 4^{2026} ?

- (A) 2^{2026} (B) 4^{1013} (C) 2^{4051} (D) 2^{1013}

■ (C): Since $4 = 2^2$, we have $(2^2)^{2026}$, or 2^{4052} . Furthermore, halving a number is equivalent to multiplying it by 2^{-1} . So one half of 4^{2026} is $2^{4052} \times 2^{-1} = 2^{4051}$.

(17) What is the last digit of 3^{2026} ?

- (A) 1 (B) 3 (C) 7 (D) 9

■ **(D):** First note that $3^4 = 81$, so squaring 3^4 must produce a number that also ends in 1: $3^4 \times 3^4 = 6561$. Since $3^4 \times 3^4 = 3^8$, it follows that for any n the last digit of 3^{4n} must be 1. The closest multiple of 4 smaller than 2026 is 2024. Thus the last digit 3^{2024} ends in 1. Since $3^{2026} = 3^{2024}3^2$, the last digit of 3^{2026} must be 9. (Using a computer it turns out that 3^{2026} has 967 digits!)

(18) How many different letter arrangements can be formed from the word SALT?

- (A) 4 (B) 6 (C) 12 (D) 24

■ **(D):** We have 4 choices for the first letter, 3 for the second, 2 for the third, and 1 for the fourth: $4! = 24$.

(19) How many different letter arrangements can be formed from the word PEPPER?

- (A) 16 (B) 24 (C) 60 (D) 720

■ **(C):** Ignoring repetitions there are $6! = 720$ possible arrangements. To deal with the repetitions, we divide 720 in half to get 360, since in all arrangements the two E's can be swapped. Similarly, the each P's can be in 3 different positions, meaning we have 6 orderings for these letters. Hence divide 360 by 6 to get 60 distinct arrangements.

(20) How many more arrangements can be made from the word MILK than the word MILL?

- (A) 2 (B) 6 (C) 12 (D) 24

■ **(C):** Both SALT and MILK have 24 possible arrangements. Since MILL has L repeated, half of the arrangements need to be discarded, leaving 12 valid arrangements, 12 fewer than for MILK.

(21) If the side length of an equilateral triangle is r , what is its height?

- (A) $\frac{r}{\sqrt{2}}$ (B) $\frac{\sqrt{3}r}{2}$ (C) $\sqrt{3}r$ (D) $\sqrt{2}r$

■ **(B):** Half the side length of the triangle is $\frac{r}{2}$. We apply the Pythagorean theorem: $h^2 + (\frac{r}{2})^2 = r^2$ gives $h^2 = \frac{3r^2}{4}$, so $h = \frac{\sqrt{3}r}{2}$.

(22) What is the external angle of a regular octagon?

- (A) 36° (B) 45° (C) 60° (D) 72°

■ **(B):** An octagon has 8 sides, so the external angle of a regular octagon is $\frac{360}{8} = 45^\circ$.

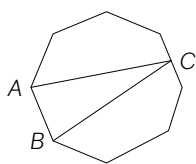


Figure 1

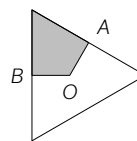


Figure 2

(23) In Figure 1 an isosceles triangle $\triangle ABC$ is constructed inside a regular octagon with a side length of 2 cm, such that A and B form one of the sides of the octagon and C is the middle of the side of the octagon opposite to AB . Which of the following is the closest to the area of $\triangle ABC$?

- (A) 3.73 cm^2 (B) 4.23 cm^2 (C) 4.83 cm^2 (D) 5.14 cm^2

■ **(C):** To get the height of $\triangle ABC$, extend AB out to a point P to create another triangle $\triangle BPD$ where D is the point on the octagon sharing a side with B that isn't A . Then $\triangle BPD$ is right angled, with two 45° angles. The hypotenuse has a side length of 2 cm, meaning the other sides have length $\sqrt{2}$ cm. Hence the height of $\triangle ABC$ is $2 + 2\sqrt{2}$ cm, and its area is $\frac{1}{2} \times 2 \times (2 + 2\sqrt{2}) = 2 + 2\sqrt{2} \text{ cm}^2$ which is closest to 4.83 cm^2 .

(24) In the equilateral triangle seen in Figure 2, A and B represent the midpoints of two sides, while O represents the triangle centre. If the side length of the triangle is 3 cm, which of the following is closest to the area of the shaded region within the triangle?

- (A) 2.60 cm^2 (B) 1.06 cm^2 (C) 1.30 cm^2 (D) 3.62 cm^2

■ **(C):** The shaded area is one third of the total area of the triangle. The height of the triangle is $\frac{3\sqrt{3}}{2}$ cm, so the area of the triangle is $\frac{9\sqrt{3}}{4} \text{ cm}^2$. Divide this by 3 to get $\frac{3\sqrt{3}}{4} \text{ cm}^2$ or 1.299 cm^2 to 3 decimal places. The closest option given is 1.30 cm^2 .

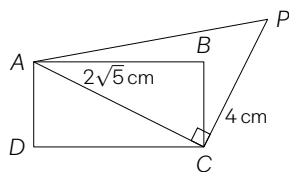


Figure 3

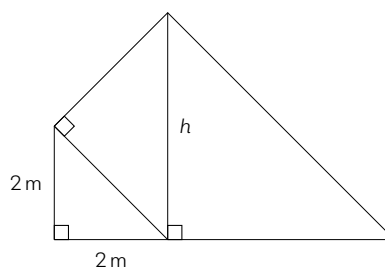


Figure 4

The next three questions refer to Figure 3.

(25) What is the distance AP ?

- (A) 4 cm (B) 6 cm (C) 8 cm (D) 36 cm

■ **(B):** We apply the Pythagorean theorem: $AP = \sqrt{4^2 + (2\sqrt{5})^2} = \sqrt{16 + 20} = 6$ cm.

(26) The rectangle $ABCD$ has perimeter 12 cm. What is the area of the rectangle $ABCD$?

- (A) 8 cm² (B) 12 cm² (C) 16 cm² (D) 24 cm²

■ **(A):** If $ABCD$ has perimeter 12 cm, then $AB + AD = 6$. Of the options available only $AD = 2$, $AB = 4$ works, giving an area of 8 cm².

(27) What is the total area of the quadrilateral $APCD$?

- (A) $4 + 4\sqrt{5}$ cm² (B) $4 + 8\sqrt{5}$ cm² (C) 16 cm² (D) $6 + 4\sqrt{5}$ cm²

■ **(A):** The triangle $\triangle ACP$ covers half the rectangle $ABCD$. Thus the area of the quadrilateral $APCD$ is equal to $\frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2\sqrt{5} \times 4 = 4 + 4\sqrt{5}$ cm².

A shape is made up of three similar right-angled triangles, as shown in Figure 4. The smallest triangle has two sides of side length 2 m.

(28) What is the area of the smallest triangle?

- (A) 1 m² (B) 2 m² (C) 4 m² (D) 6 m²

■ **(B):** $\frac{1}{2} \times 2 \times 2 = 2$ m².

(29) What is the unknown height h ?

- (A) 4 m (B) 5 m (C) 8 m (D) 16 m

■ **(A):** Apply the Pythagorean theorem to get the non-hypotenuse length j of the middle sized triangle: $j = \sqrt{2^2 + 2^2} = 2\sqrt{2}$. Apply the Pythagorean theorem again: $h = \sqrt{j^2 + j^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$ cm.

(30) What is the area of the shape?

- (A) 12 m² (B) 14 m² (C) $12 + 12\sqrt{2}$ m² (D) $24 + 20\sqrt{2}$ m²

■ **(B):** The area of the middle sized triangle is $\frac{1}{2}(2\sqrt{2})^2 = 4$ m², while the area of the largest triangle is $\frac{1}{2} \times 4 \times 4 = 8$ m². Sum the areas of the three triangles: $2 + 4 + 8 = 14$ m².

Note: the volume of a sphere is $\frac{4}{3}\pi r^3$ where r is the sphere's radius. The volume of a cylinder is $\pi r^2 h$ where r is the cylinder's radius and h is its height. The volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the cone's base and h is its height.

A cone, a sphere and a cylinder all have the same radius, r . The cylinder is 3 cm tall.

(31) What is the radius of the cylinder in terms of its volume, V ?

- (A) $r = \sqrt{\frac{V}{3\pi}}$ (B) $r = \frac{V}{3\pi}$ (C) $r = \sqrt{\frac{3\pi}{V}}$ (D) $r = \frac{3\pi}{V}$

■ **(A):** Since $V = \pi r^2 3$, we divide both sides by 3π then take the positive square root.

The sum of the volumes of the sphere and the cone is equal to the volume of the cylinder. Moreover, the sum of the heights of the sphere and the cone is equal to 2 times the height of the cylinder.

(32) What is the radius of the three shapes?

- (A) 1 cm (B) 1.5 cm (C) 2 cm (D) 2.5 cm

■ **(B):** Using the provided information we have $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \pi r^2 3$ and $2r + h = 6$. The latter can be rewritten as $h = 6 - 2r$. Putting this into the first equation we get $\frac{4}{3}\pi r^3 + 2\pi r^2 - \frac{2}{3}\pi r^3 = \pi r^2 3$, which simplifies to $\frac{2}{3}\pi r^3 = \pi r^2$. Solving this yields $r = 1.5$ cm.

(33) What is the volume of the cylinder?

- (A) 5π cm³ (B) 6.25π cm³ (C) 6.75π cm³ (D) 7.5π cm³

■ **(C):** Plugging $r = 1.5$ cm into the cylinder volume equation gives $\pi 1.5^2 3 = 6.75\pi$ cm³.

Each of the three circles in Figure 5 touch each other at one point. Their centres form a right angle triangle. The smallest circle has a radius of 1 cm, and the next smallest has a radius of 3 cm.

(34) What is the radius of the largest circle?

- (A) 2 cm (B) 4 cm (C) 6 cm (D) 7 cm

■ **(C):** The distance between the centres of the two smaller triangles is 4 cm. Let a be the distance between the centre of the largest circle and the centre of the middle circle, and b be the distance between the centre of the largest circle and the centre of the smallest circle. Then $a^2 + 4^2 = b^2$ and $a + 3 = b + 1$ (the radius of the largest circle is 3 cm more than a and also 1 cm more than b). Solving these two equations gives $a = 3$ cm and $b = 5$ cm, and in both cases this gives the radius of the largest circle as 6 cm.

(35) In degrees what is the angle θ , to one decimal place?

- (A) 47.3° (B) 53.1° (C) 64.3° (D) 72.9°

■ **(B):** We have $\sin \theta = \frac{4}{5}$. Taking the inverse sine of both sides gives $\theta = 53.1^\circ$.

(36) What is the perimeter of the shaded portion, to 1 decimal place?

- (A) 10.5 cm (B) 12.8 cm (C) 13.6 cm (D) 14.1 cm

■ **(B):** We use the arc length formula three times. The length of the perimeter portion from the largest circle is $\frac{6\theta\pi}{180} = 5.56$ cm, the length of the portion from the middle circle is $\frac{3 \times 90\pi}{180} = 4.71$ cm, and the length of the portion from the smallest circle is $\frac{1 \times (90 + \theta)\pi}{180} = 2.50$ cm. Add these together: $5.56 + 4.71 + 2.50 = 12.8$ cm to one decimal place.

(37) What is the area of the shaded portion, to 1 decimal place?

- (A) 1.5 cm² (B) 2.3 cm² (C) 2.4 cm² (D) 2.6 cm²

■ **(C):** We define the following:

- A : The area of the shaded portion.
- B : The area of the sector of the largest circle containing A .
- C : The area of the triangle defined by the centres of the three circles.
- D : The area of the sector of the middle circle inside B .
- E : The area of the sector of the smallest circle inside B .

Using these we have the equation $A = B - C - D - E$. We can calculate the values of the variables of the RHS of this equation:

- B : This is $\frac{\theta}{360} \pi 6^2 = 16.69$ cm².
- C : This is $\frac{1}{2} \times 4 \times 3 = 6$ cm².
- D : This is one quarter the area of the middle circle: $\frac{1}{4} \pi 3^2 = 7.07$ cm².
- E : This is $\frac{90 + \theta}{360} \pi 1^2 = 1.26$ cm².

Thus $A = 16.69 - 6 - 7.07 - 1.26 = 2.4$ cm² to 1 decimal place.

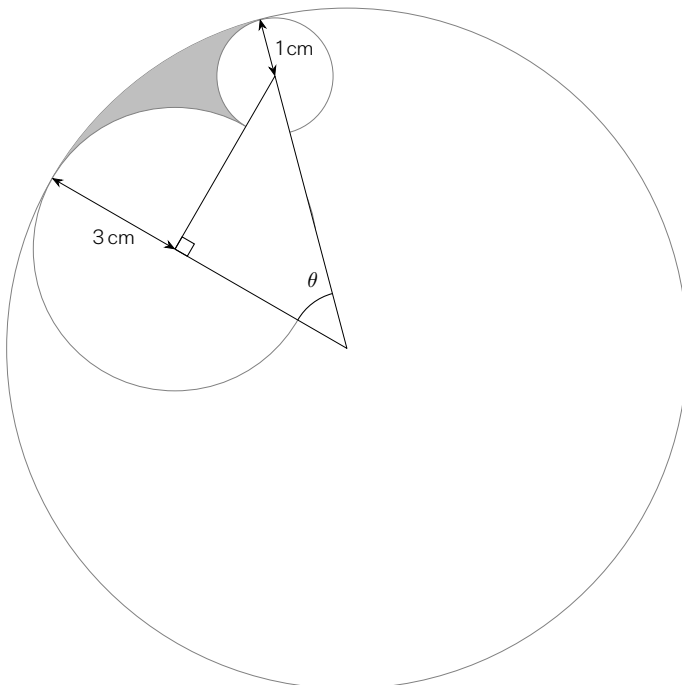


Figure 5

(70) What is the factorised form of $x^2 + x - 30$?

- (A) $(x + 5)(x - 6)$ (B) $(x - 3)(x + 10)$ (C) $(x - 5)(x + 6)$ (D) $(x + 3)(x - 10)$

■ (C): $(x - 5)(x + 6) = x^2 + 6x - 5x - 30 = x^2 + x - 30$.

(71) Which of the following are the solutions to the equation $x^2 - 5x + 5 = 1$?

- (A) $x = 2$ or $x = 3$ (B) $x = -2$ or $x = -3$ (C) $x = 4$ or $x = -1$ (D) $x = 4$ or $x = 1$

■ (D): Rearrange $x^2 - 5x + 5 = 1$ to give $x^2 - 5x + 4 = 0$. This is the same as $(x - 4)(x - 1) = 0$, so $x = 4$ or $x = 1$.

(72) How many solutions are there to the equation $(x^2 - 5x + 5)^{(x^2 + x - 30)} = 1$?

- (A) 2 (B) 3 (C) 4 (D) 6

■ (D): There are three instances where the RHS can be equal to 1: if $x^2 + x - 30 = 0$, if $x^2 - 5x + 5 = 1$, or if $x^2 - 5x + 5 = -1$ and $x^2 + x - 30$ is even.

In the first instance we can see that if $x = 5$ or $x = 6$ then $x^2 + x - 30 = 0$. Here $x^2 - 5x + 5$ equals either 5 or 11; both 5^0 and 11^0 are equal to 1.

In the second instance we have $x = 4$ or $x = 1$. Here $x^2 + x - 30$ equals either -10 or -28 ; both 1^{-10} and 1^{-28} are equal to 1.

In the final instance we have $x^2 - 5x + 6 = 0$. This is the same as $(x - 3)(x - 2)$, so $x = 3$ or $x = 2$. Then $x^2 + x - 30$ equals either -18 or -24 . Since both of these are even, -1^{-18} and -1^{24} are equal to 1.

In all instances we have two valid solutions. This means we have six valid solutions in all.