The solutions provided here are not always the only way to get to the correct answer, nor even the quickest.

In physics, the half life of a decaying substance is the time it takes to reduce to half its initial quantity. A certain radioactive substance, Zn-71, has a half-life of 2.4 minutes.

(1) How much of a 100 g initial sample of Zn-71 would be left after 7.2 minutes?

(A) 25 g (B) 12.5 g (C) 10 g (D) 6.25 g

■ **(B)**: 7.2 minutes is three half lives of Zn-71. This means we divide 100 by 2^3 to get 12.5 g.

(2) How long would it take for a 48 g sample of Zn-71 to decay to 3 g?

(A) 4.8 minutes (B) 7.2 minutes (C) 9.6 minutes (D) 12 minutes

■ **(C)**: After 2.4 minutes the sample would now weigh 24 g. After 4.8 minutes it would weigh 12 g, after 7.2 minutes 6 g, and after 9.6 minutes 3 g.

(3) An unknown quantity of the Zn-71 is placed in a box for 12 minutes. Afterwards, the remaining quantity of Zn-71 is found to weigh 5 g. How much Zn-71 was there when it was initially placed in the box?

(A) 100 g (B) 160 g (C) 320 g (D) 400 g

■ **(B)**: 12 minutes is five half lives of Zn-71. Multiply 5 by 2^5 to get 160 g.

Jed is running on a treadmill. He starts by walking at 6 km/h. After 20 minutes, he increases his speed to 12 km/h and runs for another 40 minutes.

(4) What distance did Jed travel on the treadmill in total?

(A) 10 km (B) 12 km (C) 12.8 km (D) 14 km

■ **(A)**: If Jed walked for an hour, he would walk 6 kilometres. He walks for one third of that time, so walks a distance of 2 kilometres. Similarly, he runs $\frac{2}{3} \times 12 = 8$ km. In total Jed travels $2 + 8 = 10$ km on the treadmill.

(5) What was Jed's average speed in kilometres per hour (km/h)?

(A) 8 km/h (B) 9 km/h (C) 10 km/h (D) 11 km/h

■ **(C)**: Jed only uses the treadmill for 1 hour.

Daniel is also running on a treadmill. He runs at a constant speed of 8 km/h, then slows down to a speed of 4 km/h. He runs for a total of 36 minutes, and completes a total distance of 4 km.

(6) For how long did Daniel run at 8 km/h?

(A) 12 minutes (B) 18 minutes (C) 24 minutes (D) 30 minutes

■ **(C)**: If Daniel runs at 8 km/h for 24 minutes, he covers a distance of 3.2 km. He then must run at a speed of 4 km/h for 12 minutes, a distance of 0.8 km, for a sum distance of 4 km.

(7) Daniel wants to run 2026 kilometres in 2026. He goes to the gym four times a week, and runs the same number of kilometres each time. How many kilometres should he run each time he goes to the gym if he wants to achieve his goal? Round your answer to one decimal place.

(A) 6.2 km (B) 7.3 km (C) 8.4 km (D) 9.7 km

■ **(D)**: Daniel will go to the gym $52 \times 4 = 208$ times in 2026. If he runs the same number of kilometres each time, the average time he runs will be the same as the amount he runs each time. Thus (rounded to one decimal place) he runs $\frac{2026}{208} = 9.7$ km per gym visit.

Here are a few ways of determining if a number is composite:

- Any even number that is not 2 is composite.
- Any number ending in 5 that is not 5 is composite.
- If the digits of a number greater than 3 add to a multiple of 3, that number must also be a multiple of 3, and thus is composite.

(8) Which of the following numbers is a multiple of 3?

(A) 31 (B) 36 (C) 41 (D) 46

■ **(B)**: 31 and 41 are both prime numbers, and the digits in 46 sum to 10, which is not a multiple of 3. The digits in 36 sum to 9, a multiple of 3.

(9) Which of the following numbers is a multiple of 3?

(A) 3210 (B) 2468 (C) 1234 (D) 2345

■ **(A)**: The digits in 1234 sum to 10, which is not a multiple of 3. The digits in 2345 sum to 14, which is not a multiple of 3. 2468 is twice 1234, so cannot be a multiple of 3. This leaves 3210, whose digits add to 6, a multiple of 3.

(10) Which of the following numbers is a multiple of 6?

(A) 2173 (B) 4918 (C) 5034 (D) 6183

■ **(C)**: A multiple of 6 is an even multiple of 3. This immediately excludes 6183 and 2173, which are both odd. The digits of 4918 do not sum to a multiple of 3, but the digits of 5034 do, so it must be a multiple of 6.

(11) One of the following numbers is prime. Which one is it?

(A) 111 (B) 113 (C) 115 (D) 117

■ **(B)**: 115 is a multiple of 5. The digits of 111 and 117 both sum to multiples of 3 (3 and 9 respectively). This leaves 113 as the only possible option.

A number is **polite** if it is equal to the sum of two or more consecutive positive integers. A number that cannot be written in this way is called **impolite**. For example, 3 is a polite number because it is the sum of 1 and 2, while 4 is an impolite number because it is not equal to $1+2$, $2+3$, or $1+2+3$.

The **politeness** of a number is the number of ways it can be expressed as the sum of two or more consecutive positive integers. For our examples above, 3 has a politeness of 1, while 4 has a politeness of 0.

(12) Which of the following numbers is impolite?

(A) 5 (B) 6 (C) 7 (D) 8

■ **(D)**: $5 = 2+3$, $6 = 1+2+3$, and $7 = 3+4$, so all these numbers must all be polite. Thus 8 is an impolite number.

(13) What is the politeness of 9?

(A) 0 (B) 1 (C) 2 (D) 3

■ **(C)**: Since $9 = 4+5$ and $9 = 2+3+4$, the politeness of 9 is 2.

(14) Which of the following numbers has the largest politeness?

(A) 26 (B) 27 (C) 28 (D) 29

■ **(B)**: 26 has a politeness of 1: $5+6+7+8 = 26$. 27 has a politeness of 3: $13+14 = 27$, $8+9+10 = 27$, and $2+3+4+5+6+7 = 27$. Like 26, 28 has a politeness of 1: $1+2+3+4+5+6+7 = 28$. Finally 29 also has a politeness of 1: $14+15 = 29$. Thus 27 has the largest politeness.

A cylindrical water tank is leaking water at a constant rate. In two minutes, the height of the water in the tank drops by 15 cm. In the same amount of time, the water that escaped exactly fills a 50 L container. (1 L = 1000 cm^3.)

(15) What is the rate of decrease of the height of the water in meters per second? Give your answer to the nearest 2 significant figures.

(A) 0.15 m/s (B) 0.0125 m/s (C) 0.013 m/s (D) 0.0013 m/s

■ **(D)**: Since 15 cm $=$ 0.15 m and 2 minutes is 120 seconds, the rate of decrease is $\frac{0.15}{120} = 0.00125\,\mathrm{m/s}$ which to 2 significant figures is 0.0013 m/s.

(16) What is the rate of flow of the water out of the tank in cm^3/s? Give your answer to the nearest 2 significant figures.

(A) 430 cm^3/s (B) 420 cm^3/s (C) 416 cm^3/s (D) 410 cm^3/s

■ **(B)**: Since $50\,\mathrm{L} = 50000\,\mathrm{cm}^3$, the rate of flow of the water out of the tank is $\frac{50000}{120} = 416.67\,\mathrm{cm}^3/\mathrm{s}$ to 2 decimal places. To 2 significant figures this value is 420 cm^3/s.

(17) What is the radius of the tank in cm? Give your answer to the nearest 2 significant figures.

(A) 33 cm (B) 32 cm (C) 28 cm (D) 25 cm

■ **(A)**: The volume of a cylinder is found using the formula $\pi r^2 h$. We have $15 \times \pi r^2 = 50000$. Thus $r^2 = 1061.03$ to 2 decimal places, and so $r = 32.57$ cm to 2 decimal places. To 2 significant figures this value is 33 cm.

(18) The tank is 120 cm tall. What is the total capacity of the tank?

(A) 150 L (B) 200 L (C) 300 L (D) 400 L

■ **(D)**: Since $15 \times 8 = 120$, the capacity is $8 \times 50 = 400$ L.

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(19) What is the probability that the sum of two six-sided dice rolls is exactly 4?

(A) $1/12$ (B) $1/9$ (C) $1/6$ (D) $1/3$

■ **(A)**: There are 36 possible dice rolls. Of these 3 sum to 4: 1 + 3, 3 + 1, and 2 + 2. Thus the probability is $\frac{3}{36} = \frac{1}{12}$.

(20) What is the probability that after three six-sided dice rolls neither a 2 nor a 6 is rolled?

(A) $64/216$ (B) $125/216$ (C) $152/216$ (D) $91/216$

■ **(A)**: The probability of not rolling a 2 or a 6 on a single die roll is $\frac{4}{6} = \frac{2}{3}$. This means the probability of not rolling a 2 or a 6 after three dice rolls is $(\frac{2}{3})^3 = \frac{8}{27}$. Since $27 \times 8 = 216$ this is equivalent to $\frac{64}{216}$.

(21) Three consecutive one digit integers are a perfect square, a perfect cube and a prime (not necessarily in that order). What is the product of these three integers?

(A) 60 (B) 210 (C) 336 (D) 504

■ **(D)**: The only perfect cubes with one digit are 1 and 8. Since neither 2 nor 3 are perfect squares, it follows that the perfect cube must be 8. This works, as 9 is a perfect square and 7 is prime. Take the product of the three: $7 \times 8 \times 9 = 504$.

(22) What is $\dfrac{2^8}{8^2}$ equal to?

(A) 2 (B) 4 (C) 16 (D) 64

■ **(B)**: Since $8 = 2^3$, we have $\frac{2^8}{(2^3)^2} = \frac{2^8}{2^6}$. Simplifying we get 2^2 or 4.

(23) What is half of 4^{2026}?

(A) 2^{2026} (B) 4^{1013} (C) 2^{4051} (D) 2^{1013}

■ **(C)**: Since $4 = 2^2$, we have $(2^2)^{2026}$, or 2^{4052}. Furthermore, halving a number is equivalent to multiplying it by 2^{-1}. So one half of 4^{2026} is $2^{4052} \times 2^{-1} = 2^{4051}$.

(24) What is the last digit of 3^{2026}?

(A) 1 (B) 3 (C) 7 (D) 9

■ **(D)**: First note that $3^4 = 81$, so squaring 3^4 must produce a number that also ends in 1: $3^4 \times 3^4 = 6561$. Since $3^4 \times 3^4 = 3^8$, it follows that for any n the last digit of 3^{4n} must be 1. The closest multiple of 4 smaller than 2026 is 2024. Thus the last digit 3^{2024} ends in 1. Since $3^{2026} = 3^{2024}3^2$, the last digit of 3^{2026} must be 9. (Using a computer it turns out that 3^{2026} has 967 digits!)

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(25) How many different letter arrangements can be formed from the word SALT?

(A) 4 (B) 6 (C) 12 (D) 24

■ **(D)**: We have 4 choices for the first letter, 3 for the second, 2 for the third, and 1 for the fourth: $4! = 24$.

(26) How many different letter arrangements can be formed from the word PEPPER?

(A) 16 (B) 24 (C) 60 (D) 720

■ **(C)**: Ignoring repetitions there are $6! = 720$ possible arrangements. To deal with the repetitions, we divide 720 in half to get 360, since in all arrangements the two E's can be swapped. Similarly, the each P's can be in 3 different positions, meaning we have 6 orderings for these letters. Hence divide 360 by 6 to get 60 distinct arrangements.

(27) How many more arrangements can be made from the word MILK than the word MILL?

(A) 2 (B) 6 (C) 12 (D) 24

■ **(C)**: Both SALT and MILK have 24 possible arrangements. Since MILL has L repeated, half of the arrangements need to be discarded, leaving 12 valid arrangements, 12 fewer than for MILK.

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(28) If the side length of an equilateral triangle is r, what is its height?

(A) $\frac{r}{\sqrt{2}}$ (B) $\frac{\sqrt{3}r}{2}$ (C) $\sqrt{3}r$ (D) $\sqrt{2}r$

■ **(B)**: Half the side length of the triangle is $\frac{r}{2}$. We apply the Pythagorean theorem: $h^2 + (\frac{r}{2})^2 = r^2$ gives $h^2 = \frac{3r^2}{4}$, so $h = \frac{\sqrt{3}r}{2}$.

(29) What is the external angle of a regular octagon?

(A) 36° (B) 45° (C) 60° (D) 72°

■ **(B)**: An octagon has 8 sides, so the external angle of a regular octagon is $\frac{360}{8} = 45°$.

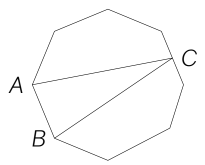

Figure 1

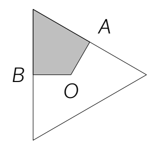

Figure 2

(30) In Figure 1 an isosceles triangle $\triangle ABC$ is constructed inside a regular octagon with a side length of 2 cm, such that A and B form one of the sides of the octagon and C is the middle of the side of the octagon opposite to AB. Which of the following is the closest to the area of $\triangle ABC$?

(A) $3.73\,\text{cm}^2$ (B) $4.23\,\text{cm}^2$ (C) $4.83\,\text{cm}^2$ (D) $5.14\,\text{cm}^2$

■ **(C)**: To get the height of $\triangle ABC$, extend AB out to a point P to create another triangle $\triangle BPD$ where D is the point on the octagon sharing a side with B that isn't A. Then $\triangle BPD$ is right angled, with two 45° angles. The hypotenuse has a side length of 2 cm, meaning the other sides have length $\sqrt{2}$ cm. Hence the height of $\triangle ABC$ is $2 + 2\sqrt{2}$ cm, and its area is $\frac{1}{2} \times 2 \times (2 + 2\sqrt{2}) = 2 + 2\sqrt{2}\,\text{cm}^2$ which is closest to $4.83\,\text{cm}^2$.

(31) In the equilateral triangle seen in Figure 2, A and B represent the midpoints of two sides, while O represents the triangle centre. If the side length of the triangle is 3 cm, which of the following is closest to the area of the shaded region within the triangle?

(A) $2.60\,\text{cm}^2$ (B) $1.06\,\text{cm}^2$ (C) $1.30\,\text{cm}^2$ (D) $3.62\,\text{cm}^2$

■ **(C)**: The shaded area is one third of the total area of the triangle. The height of the triangle is $\frac{3\sqrt{3}}{2}$ cm, so the area of the triangle is $\frac{9\sqrt{3}}{4}\,\text{cm}^2$. Divide this by 3 to get $\frac{3\sqrt{3}}{4}\,\text{cm}^2$ or $1.299\,\text{cm}^2$ to 3 decimal places. The closest option given is $1.30\,\text{cm}^2$.

Note: the volume of a sphere is $\frac{4}{3}\pi r^3$ where r is the sphere's radius. The volume of a cylinder is $\pi r^2 h$ where r is the cylinder's radius and h is its height. The volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the cone's base and h is its height.

A cone, a sphere and a cylinder all have the same radius, r. The cylinder is 3 cm tall.

(32) What is the radius of the cylinder in terms of its volume, V?

(A) $r = \sqrt{\frac{V}{3\pi}}$ (B) $r = \frac{V}{3\pi}$ (C) $r = \sqrt{\frac{3\pi}{V}}$ (D) $r = \frac{3\pi}{V}$

■ **(A)**: Since $V = \pi r^2 3$, we divide both sides by 3π then take the positive square root.

The sum of the volumes of the sphere and the cone is equal to the volume of the cylinder. Moreover, the sum of the heights of the sphere and the cone is equal to 2 times the height of the cylinder.

(33) What is the radius of the three shapes?

(A) 1 cm (B) 1.5 cm (C) 2 cm (D) 2.5 cm

■ **(B)**: Using the provided information we have $\frac{4}{3}\pi r^3 + \frac{1}{3}\pi r^2 h = \pi r^2 3$ and $2r + h = 6$. The latter can be rewritten as $h = 6 - 2r$. Putting this into the first equation we get $\frac{4}{3}\pi r^3 + 2\pi r^2 - \frac{2}{3}\pi r^3 = \pi r^2 3$, which simplifies to $\frac{2}{3}\pi r^3 = \pi r^2$. Solving this yields $r = 1.5$ cm.

(34) What is the volume of the cylinder?

(A) 5π cm^3 (B) 6.25π cm^3 (C) 6.75π cm^3 (D) 7.5π cm^3

■ **(C)**: Plugging $r = 1.5$ cm into the cylinder volume equation gives $\pi 1.5^2 3 = 6.75\pi$ cm^3.

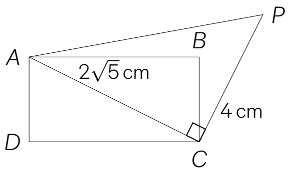

Figure 3

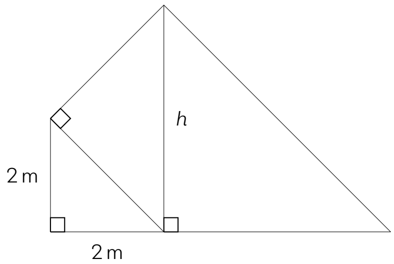

Figure 4

The next three questions refer to Figure 3.

(35) What is the distance AP?

(A) 4 cm (B) 6 cm (C) 8 cm (D) 36 cm

■ **(B)**: We apply the Pythagorean theorem: $AP = \sqrt{4^2 + (2\sqrt{5})^2} = \sqrt{16 + 20} = 6$ cm.

(36) The rectangle $ABCD$ has perimeter 12 cm. What is the area of the rectangle $ABCD$?

(A) 8 cm^2 (B) 12 cm^2 (C) 16 cm^2 (D) 24 cm^2

■ **(A)**: If $ABCD$ has perimeter 12 cm, then $AB + AD = 6$. Of the options available only $AD = 2$, $AB = 4$ works, giving an area of 8 cm^2.

(37) What is the total area of the quadrilateral $APCD$?

(A) $4 + 4\sqrt{5}$ cm^2 (B) $4 + 8\sqrt{5}$ cm^2 (C) 16 cm^2 (D) $6 + 4\sqrt{5}$ cm^2

■ **(A)**: The triangle $\triangle ACP$ covers half the rectangle $ABCD$. Thus the area of the quadrilateral $APCD$ is equal to $\frac{1}{2} \times 2 \times 4 + \frac{1}{2} \times 2\sqrt{5} \times 4 = 4 + 4\sqrt{5}$ cm^2.

A shape is made up of three similar right-angled triangles, as shown in Figure 4. The smallest triangle has two sides of side length 2 m.

(38) What is the area of the smallest triangle?

(A) 1 m^2 (B) 2 m^2 (C) 4 m^2 (D) 6 m^2

■ **(B)**: $\frac{1}{2} \times 2 \times 2 = 2$ m^2.

(39) What is the unknown height h?

(A) 4 m (B) 5 m (C) 8 m (D) 16 m

■ **(A)**: Apply the Pythagorean theorem to get the non-hypotenuse length j of the middle sized triangle: $j = \sqrt{2^2 + 2^2} = 2\sqrt{2}$. Apply the Pythagorean theorem again: $h = \sqrt{j^2 + j^2} = \sqrt{(2\sqrt{2})^2 + (2\sqrt{2})^2} = \sqrt{16} = 4$ cm.

(40) What is the area of the shape?

(A) 12 m^2 (B) 14 m^2 (C) $12 + 12\sqrt{2}$ m^2 (D) $24 + 20\sqrt{2}$ m^2

■ **(B)**: The area of the middle sized triangle is $\frac{1}{2}(2\sqrt{2})^2 = 4$ m^2, while the area of the largest triangle is $\frac{1}{2} \times 4 \times 4 = 8$ m^2. Sum the areas of the three triangles: $2 + 4 + 8 = 14$ m^2.