The University of Otago
Junior Mathematics Competition
2025 Student Report

Department of Mathematics and Statistics   Te Tari Pākarau me te Tatauraka

University of Otago
ŌTĀKOU WHAKAIHU WAKA

Year 9 Prize Winners

First Doris Xiao, Pinehurst School
Second Jasper Li, St Kentigern College
Third Jaehwan Kim, Aquinas College

Top 30 (in School Order):

Seunghye Chang, ACG Parnell College
Connor Hu-Wen, Macleans College
Raymont Qin, Macleans College
Ivo Zheng, Macleans College
Leo Liu, Pinehurst School
Xujia Zhang, Rangi Ruru Girls' School
Ryan Lau, Rangitoto College
Emma Zhang, Rangitoto College
Anna Phillipps, St Peter's School (Cambridge)
Samuel Martin, Burnside High School
Stella Kim, Macleans College
Elliot Wang, Macleans College
Muhammad Mangera, Newlands College
Felix Liu, Pinehurst School
Yiyang Cao, Rangitoto College
Jonathan Xu, Rangitoto College
Sam Li, St Andrew's College
Charles Mao, Wentworth College
Max Tai, Whitby Collegiate
Max Yang, King's College
Ethan Liu, Macleans College
Oscar Zhang, Macleans College
Bosco Jin, Pinehurst School
Kevin Shen, Pinehurst School
Daniel Fang, Rangitoto College
Judy Yu, Rangitoto College
Benjamin Yu, St Andrew's College
J K Wang, Wentworth College

Year 10 Prize Winners

First Roy Luo, Macleans College
Second Sophie Xu, Kristin School
Third Vani Singh, Epsom Girls' Grammar School

Top 30 (in School Order):

Haoen Cao, ACG Parnell College
Johnny Huang, Burnside High School
Asher Goh, Glendowie College
Benjamin Chai, Macleans College
Eric Shen, Macleans College
Amelia Baudinet, Marist College
Matthew Sun, Pinehurst School
Brendon Jang, Rangitoto College
Ruishan Bella Chen, St Kentigern College
Isabelle Shi, ACG Parnell College
Mercy Song, Burnside High School
Isabel Oomens, Glendowie College
Isaac Chen, Macleans College
Jeffrey Wang, Macleans College
James Patterson, Paraparaumu College
Gina Xiang, Pinehurst School
Siwoo Jang, Rangitoto College
Lisa Murata Gutierrez, Takapuna Grammar School
Daniel Noronha, Avondale College
Ivy Huang, Epsom Girls' Grammar School
Luo Yuan Bao, Macleans College
Heidi Lee, Macleans College
Zhiyuan Yu, Macleans College
Zion Feng, Pinehurst School
Joshua Zhang, Pinehurst School
Mickey Zhao, St Andrew's College
Matthew Wu, Westlake Boys' High School

Year 11 Prize Winners

First George Zhao, Botany Downs Secondary College
Second Jackie Xu, St Cuthbert's College
Third Eric Liu, Macleans College

Top 30 (in School Order):

Cheng Yu Li, ACG Parnell College
Alvin Ting, Burnside High School
Shrika Nitsingham, Epsom Girls' Grammar School
Isabelle Ning, Kristin School
Isla Wang, Macleans College
Alston Huang, Rangitoto College
Jimmy Zhou, Rangitoto College
Rico McWilliams, St Peter's College (Epsom)
Doyoon Kwak, Tauranga Boys' College
Alan Li, Bethlehem College
Leon Wu, Burnside High School
Rin Lott, Green Bay High School
Josephine Chong, Macleans College
Lucy Zhang, Macleans College
Felix Luo, Rangitoto College
Summer Li, St Cuthbert's College
Daniel Sheldon, St Peter's College (Epsom)
Bella McLauchlan-Hillary, Waikato Diocesan School for Girls
Edward Cheng, Burnside High School
Jasmine Chen, Carmel College
Harrison Wong, King's College
Daniel Lee, Macleans College
Yicheng Wang, Pinehurst School
Ying Yuan Zhang, Rangitoto College
Emma Zheng, St Cuthbert's College
Shia Abdul-Coley, Tauranga Boys' College
Xiaoyang Shawn Zhou, Westlake Boys' High School

As always only one method is shown. Often several methods exist. We don't guarantee that any method shown is the best or fastest in any case.

Question 1: 10 marks (Years 9 and below only)

A 500 mL bottle of whole milk has the following nutritional values per 100 mL:

Nutrient	Amount per 100 mL
Protein	3.3 g
Fat	3.4 g
Carbohydrates	4.8 g

(a) Based on the values per 100 mL, calculate the total amount of protein, fat, and carbohydrates in the entire 500 mL bottle of milk.

■ **Protein:** $3.3 \times 5 = 16.5\,\text{g}$

Fat: $3.4 \times 5 = 17\,\text{g}$

Carbohydrates: $4.8 \times 5 = 24\,\text{g}$

(b) Given that protein and carbohydrates provide 4 kcal per gram and fat provides 9 kcal per gram, determine how many calories (in kcal) come from each nutrient in the 500 mL bottle.

■ **Protein:** $16.5 \times 4 = 66\,\text{kcal}$

Fat: $17 \times 9 = 153\,\text{kcal}$

Carbohydrates: $24 \times 4 = 96\,\text{kcal}$

(c) Calculate the total amount of calories (in kcal) from the three nutrients in the 500 mL bottle of milk. Determine what percentage of the total amount of calories comes from fat.

■ The total calories is $66 + 153 + 96 = 315\,\text{kcal}$. The percentage of the total amount of calories that comes from fat is then $\frac{153}{315} \times 100\% \approx 48.6\%$.

This was done well overall, with an average grade of over 50%. Some students did not round their percentage in part (c) to a sensible amount (one or two decimal places is suggested). You shouldn't always write down exactly what your calculator displays!

Question 2: 10 marks (Years 10 and below only)

Local honey producers process and package Manuka honey before selling it in the market. Three types of packaging, Large, Medium, and Small, are available with the following details:

Package Type	Weight (g per jar)	Production Cost ($ per jar)	Selling Price ($ per jar)
Large	400	35	45
Medium	300	28	36
Small	200	19	25

If each type of packaging has sold 120 kg of Manuka honey:

(a) Calculate the total number of jars sold for each package type.

■ **Large:** $120\,000 \div 400 = 300$ jars

Medium: $120\,000 \div 300 = 400$ jars

Small: $120\,000 \div 200 = 600$ jars

(b) Which type of packaging contributes the most profit, and what is the total profit obtained for that type?

■ The small jars provide the most profit:

Large: $45 - 35 = 10$, and $10 \times 300 = \$3000$

Medium: $36 - 28 = 8$, and $8 \times 400 = \$3200$

Small: $25 - 19 = 6$, and $6 \times 600 = \$3600$

(c) Quinn wants to buy 600g of Manuka honey. What is the cheapest way for Quinn to purchase this amount?

■ Quinn should buy 1 small jar and 1 large jar:

3 Small Jars: $(3 \times 200\,\text{g} = 600\,\text{g}.)$ $3 \times 25 = \$75$

2 Medium Jars: $(2 \times 300\,\text{g} = 600\,\text{g}.)$ $2 \times 36 = \$72$

1 Small Jar and 1 Large Jar: $(200\,\text{g} + 400\,\text{g} = 600\,\text{g}.)$ $45 + 25 = \$70$

(No other combination of jars is possible.)

Like question one this was well done for the most part. In (b), some students did not know how to calculate the profit for each jar type, while others forgot to multiple the profit per jar by the number of jars. In (c), some students tried combinations of jars that were not possible, like one and a half large jars, or forgot to check one of the jar types entirely.

Question 3: 20 marks (All Years)

If p, q, r are three consecutive prime numbers, q is a **balanced prime** if it is the mean of the two neighboring prime numbers p and r. For example, $5 = (3 + 7) \div 2$ and $157 = (151 + 163) \div 2$ are both balanced primes.

(a) Which of the following numbers is a balanced prime number?

51, 53, 57, 59

■ Both 51 and 57 are multiples of 3, and thus are not prime.

59 is prime. The primes before and after 59 are 53 and 61, but the mean of these two primes is 57, so 59 is not a balanced prime.

That leaves 53, which like 59 is prime. The primes before and after 53 are 47 and 59, and the mean of these two primes is 53, thus 53 is a balanced prime.

(b) Check whether 97 is a balanced prime number.

■ The primes before and after 97 are 89 and 101. The mean of these two primes is 95, not 97, thus 97 is not a balanced prime number.

(c) What is the smallest balanced prime number above 157?

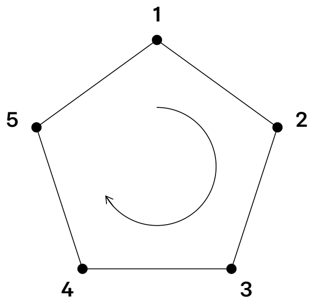
■ We have to test each prime above 157 in turn until we find one that is also a balanced prime:

- Above we see that 163 is the smallest prime larger than 157. The next smallest prime is 167. However the mean of 157 and 167 is 162 and not 163, thus 163 is not a balanced prime.
- We now check 167. The next smallest prime is 173. Here the mean of 163 and 173 is 168 and not 167, thus 167 is not a balanced prime.
- Next is 173. The next smallest prime is 179. This time the mean of 167 and 179 is 173, so 173 is the smallest balanced prime larger than 157.

This was not done well overall. In (a) many students failed to notice that 51 and 57 are not prime, which meant their checking of both 53 and 59 was also flawed. Part (b) was better done overall, although quite a few students used 91 (or other composite numbers) rather than 89, leading them to the correct conclusion from faulty origins. Although quite a few students in (c) could identify 173 as the smallest balanced prime larger than 157, they did not demonstrate that 163 and 167 were not balanced prime numbers. For full credit in questions like this, not only must the right number be identified, but other possiblities must be excluded.

Question 4: 20 marks (All Years)

Cayley is playing a game in a pentagon-shaped park where the corners are labelled 1, 2, 3, 4, and 5. If Cayley starts at one corner and walks along the edges in a clockwise direction, the number of steps he takes corresponds to the label of the corner where he started. This walking method is referred to as a "shift".
For example, if Cayley starts at corner 3, he would walk 3 steps: from corner $3 \to 4 \to 5 \to 1$. After the first "shift", he ends up at corner 1. Then, starting from corner 1, he would walk 1 step: from corner $1 \to 2$, completing the second "shift".



If Cayley starts at corner 2, answer the following questions:

(a) What is the corner number he reaches after 2 shifts?

- First, Cayley takes 2 steps clockwise:

$$2 \to 3 \to 4$$

Then, he takes 4 further steps:

$$4 \to 5 \to 1 \to 2 \to 3$$

After 2 shifts, Cayley is at corner 3.

(b) What is the corner number he reaches after 4 shifts?

- As seen above, Cayley starts the third shift at corner 3. He then takes 3 steps clockwise:

$$3 \to 4 \to 5 \to 1$$

Then, he takes 1 further step:

$$1 \to 2$$

After 4 shifts, Cayley is at corner 2.

(c) What is the corner number he reaches after 10 shifts?

- As seen above, Cayley starts the fifth shift at corner 2. So:

Shift 5: $2 \to 3 \to 4$.

Shift 6: $4 \to 5 \to 1 \to 2 \to 3$.

Shift 7: $3 \to 4 \to 5 \to 1$.

Shift 8: $1 \to 2$.

Shift 9: $2 \to 3 \to 4$.

Shift 10: $4 \to 5 \to 1 \to 2 \to 3$.

After 10 shifts, Cayley is at corner 3.

To celebrate the arrival of the year 2025, Cayley makes a heroic move, starting at corner 1 and completing 2025 shifts in the pentagon-shaped park. Answer the following question:

(d) What is the corner number he reaches?

- From the first three parts to the question we can see that only certain 'patterns' of movement are possible. This suggests a cycle of some sort. We can find this cycle by looking at the first 8 shifts Cayley must take in his heroic move:

Shift 0: 1.

Shift 1: $1 + 1 = 2$.

Shift 2: $2 + 2 = 4$.

Shift 3: $4 + 4 = 8 \equiv 3 \bmod 5$.

Shift 4: $3 + 3 = 6 \equiv 1 \bmod 5$.

Shift 5: $1 + 1 = 2$.

Shift 6: $2 + 2 = 4$.

Shift 7: $4 + 4 = 8 \equiv 3 \bmod 5$.

Shift 8: $3 + 3 = 6 \equiv 1 \bmod 5$.

It is clear there is a cycle of length 4: $1 \to 2 \to 4 \to 3 \to 1$.

Since Cayley starts at corner 1, after 4 shifts he will return to that corner. So for 2025 shifts:

$$2025 \equiv 1 \bmod 4$$

Hence Cayley will have effectively only done 1 shift, which means he ends on corner 2.

This was the best answered Question 4 in many years. Many students grasped the concept immediately, and correct answers for the first three answers were relatively common. What was often missed here was a mathematical rigour, especially in part (d) where students could come up with the correct answer, but could not describe how in a clear and logical way.

Question 5: 20 marks (All Years)

In a numerical transformation game, we define the numbers 0, 1, 2, …, 50 as "old numbers". To transform an old number, the following steps are used:

(1) Square the old number.

(2) Divide the result by 100.

(3) The remainder is the "new number".

Example: For the old number 39,

$$39^2 = 1521, \quad 1521 \div 100 = 15 \text{ with remainder } 21$$

So, the new number for 39 is 21.

(a) Apply this rule to transform 26 into its corresponding new number.

- The new number is 76:

$$26^2 = 676$$
$$676 \equiv 76 \bmod 100.$$

(b) Given a new number 36, find its corresponding old number(s).

- We want x such that $x^2 \equiv 36 \bmod 100$ (the remainder of x^2 after dividing by 100 is 36).

It shouldn't be too hard to see that $x = 6$ works here. To see if there any others, first note for a number when squared will end in a 6 if that number ended in 4 or 6. This means we need to test 4, 14, 16, 24, 26, 34, 36, 44, and 46. Of these only 44 works: $44^2 = 1936$, which when by divided by 100 has remainder 36 as required.

(c) After transformation, many numbers decrease in value. Jacob claims, "No new number is equal to its old number". Do you agree with Jacob? If not, find all old numbers for which the new number remains the same.

- Suppose x is an old number that doesn't change when transformed. Then

$$x^2 \equiv x \bmod 100$$
$$x^2 - x \equiv 0 \bmod 100$$
$$x(x-1) = 100k$$

If $k = 0$ then both $x = 0$ and $x = 1$ satisfy the above equation.

Now suppose $k > 0$. Both sides of the above equation have 25 as a factor, so at most one of x or $x - 1$ must have 25 as a factor. Hence either $x = 25$, $x = 26$, or $x = 50$. Neither 26×25 nor 50×49 have 100 as a factor. 25×24 does however, so check 25: $25^2 = 625$, which has 25 as a remainder after division by 100. Hence 25 also stays the same.

Overall we have found 3 old numbers that do not change after transformation: 0, 1, and 25. Thus Jacob is incorrect.

(d) Identify the old number that decreases the most after transformation and show your working.

■ Since $50^2 = 2500$ which has remainder 0 after division by 100, 50 decreases by 50 after transformation. Any other old number must decrease by less after transformation.

In some ways student performance was similar to that in Question 4, albeit with a lower average success rate. For those who understood the number transformation process, parts (a) and (b) were particularly well done. Most students who could get correct answers in parts (b) and (c) could not explain exactly how they got their answers, and frequently neglected to check that they were the only possible answers. Brute force was common compared to more elegant methods; while the former method often produced the correct result, the time taken left students employing it less time to complete the remaining questions.

Question 6: 20 marks (All Years)

A vending machine offers three types of snacks: chips, chocolate, and cookies. Customers can select snacks based on their preferences.

(a) In how many ways can a customer choose exactly two different snacks from the three available options?

■ Choose 2 different snacks from 3 options: $\binom{3}{2} = 3$.

Later, the vending machine adds a fourth snack option: juice, providing more choices.

(b) How many ways can a customer now select exactly two different snacks from the four available options?

■ Choose 2 different snacks from 4 options: $\binom{4}{2} = 6$.

(c) If a customer can choose any two snacks, but they don't have to be different (i.e., they can pick two of the same snack), how many different selections are possible?

■ From (b) we had 6 ways. Now there are 4 additional ways (two chips, two chocolates, two cookies, and two juices). Thus there are now 10 ways we can select two snacks.

(d) If the vending machine limits each customer to a maximum of three snacks per purchase (they don't have to be different), how many different ways can a customer select their snacks from the four options?

■ A customer can buy no snacks, one snack, two snacks, or three snacks. There is only 1 way you can buy no snacks, and 4 ways you can buy one snack.

If two snacks are to be bought, first suppose one of the snacks is chocolate. Then there are 4 options for the other snack. Next suppose that chocolate is not bought at all, but cookies were. Then there are 3 options for the other snack. It follows that there must be 2 ways to buy the snacks if neither chocolate nor cookies were bought (but chips were), and 1 way if only one snack (juice) is available. This is a total of 10 ways snacks can be bought when we buy two snacks of any type.

In a similar way there are 20 ways of buying three snacks — 10 ways when chocolate is bought, 6 ways when chocolate is not bought but cookies are, 3 ways when neither chocolate or cookies are bought, and 1 way if only one snack is available.

In total there are $1 + 4 + 10 + 20 = 35$ ways of buying any combination of up to three snacks.

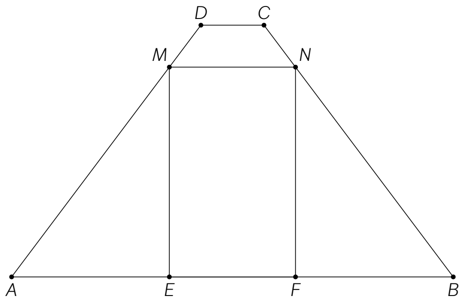
This is an uncomplicated but not easy question. The use of combinations in parts (a) and (b) were common, although brute force was also common (and in this case not a huge waste of time). Many students were quick to use their answer in (b) to produce their answer to (c), which was pleasing to see. Part (d), the hardest part of the question proved the downfall of almost all students in the competition, however — of those who made the most progress, only a handful realised that not taking a snack was an option, meaning full marks in this question was a rare occurrence.

Question 7: 10 marks (Years 10 and 11 only)

In the trapezium $ABCD$, $AB \parallel CD$, where:

$$AB = 7, \quad CD = 1, \quad AD = BC = 5$$

Points M and N are on AD and BC, respectively. We have $MN \parallel AB$, $ME \perp AB$ and $NF \perp AB$, with E and F being the respective foot points.



(a) Calculate the area of the trapezium $ABCD$.

■ Add points G and H to AB such that CH and DG are both perpendicular to AB. Then $\triangle CHB$ is a right angled triangle, with an hypotenuse of length 5.

Since $AD = BC$ and $CD \parallel AB$, $ABCD$ must be an isosceles trapezium. Thus the length of BH is half the difference between the length AB and the length of CD: $(7-1)/2 = 3$. We have a right angled triangle with an hypotenuse of length 5 and another side of length 3, so the final side must have length 4 by Pythagoras.

We then use the standard trapezium area formula: $\frac{1}{2}(AB + CD) \times h = \frac{1}{2}(7+1) \times 4 = 16$.

(b) Express the area of the quadrilateral $MEFN$ in terms of x, where $AE = x$.

■ If $AE = x$, then since $AE = FB$ we have $MN = AB - 2x = 7 - 2x$.

Using our defined point G from (a), we see that $\triangle AEM$ and $\triangle AGD$ are similar. Thus $\frac{x}{3} = \frac{ME}{4}$, so $ME = \frac{4x}{3}$. This means that the area of $MEFN$ is $(7-2x) \cdot \frac{4x}{3} = \frac{28x - 8x^2}{3}$.

(c) If points M and N can move along AD and BC, respectively, while always ensuring $MN \parallel AB$, determine whether the quadrilateral $MEFN$ can be a square. If yes, find its area; if not, explain why.

■ For $MEFN$ to be a square, we require $ME = MN$. From (b), $ME = \frac{4x}{3}$ and $MN = 7 - 2x$. Equating the two we get $\frac{4x}{3} = 7 - 2x$ or $4x = 21 - 6x$. Solving, x has a non-negative value of $\frac{21}{10}$, so $MEFN$ can be a square.

Using our value of x, we see that $ME = \frac{4 \cdot 21}{3 \cdot 10} = \frac{14}{5}$, and so the area of $MEFN$ is $\left(\frac{14}{5}\right)^2 = \frac{196}{25}$.

The easier of the two geometry questions in the 2025 competition was nonetheless too difficult for the majority of students. Many of those who attempted the question could get part (a) correct at least. Part (b) proved to be too hard for many students. Those who could get part (b) correct would normally also get part (c) correct. On the whole students did about as well here as the equivalent question in the 2024 competition.

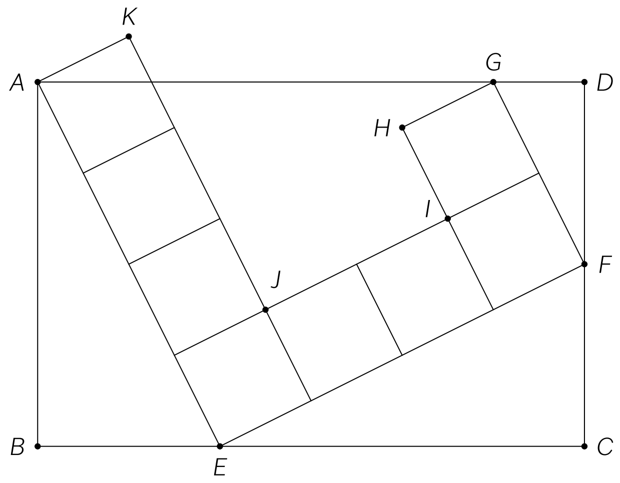
Question 8: 10 marks (Year 11 only)

In the given diagram, we have a rectangle $ABCD$ and a polygon $AEFGHIJK$ composed of 8 identical small squares, each with an area of 1 unit. Points E, F and G are on the perimeter of the rectangle $ABCD$.

Determine the perimeter of the rectangle $ABCD$.

■ From the given relationships in the diagram and geometric reasoning:

- $\angle B = \angle C = \angle D = 90°$.
- From the given configuration, $AE = EF = 4$ and $FG = 2$.
- $\angle BAE = \angle CEF = \angle DFG$ since they are complementary angles of right angles.
- Thus, $\triangle ABE \cong \triangle ECF$ and $\triangle ECF \sim \triangle FDG$ by angle-angle (AA) similarity.



- Let $BE = x$. Then by congruence and similarity:

$$AB = 2x,$$
$$CE = AB = 2x,$$
$$DF : CE = FG : EF = 2 : 4 = 1 : 2 \Rightarrow DF = x.$$

- Using the Pythagorean Theorem in $\triangle ABE$:

$$AB^2 + BE^2 = AE^2$$
$$(2x)^2 + x^2 = 4^2$$
$$4x^2 + x^2 = 16$$
$$5x^2 = 16$$
$$x^2 = \frac{16}{5}, \quad x = \frac{4}{\sqrt{5}} = \frac{4\sqrt{5}}{5}$$

- Now, to find the side lengths of the rectangle:

$$AB = 2x = 2 \cdot \frac{4\sqrt{5}}{5} = \frac{8\sqrt{5}}{5}$$
$$AD = 3x = 3 \cdot \frac{4\sqrt{5}}{5} = \frac{12\sqrt{5}}{5}$$

- Therefore, the perimeter is:

$$2(AB + AD) = 2\left(\frac{8\sqrt{5}}{5} + \frac{12\sqrt{5}}{5}\right) = 2 \cdot \frac{20\sqrt{5}}{5} = 2 \cdot 4\sqrt{5} = 8\sqrt{5}$$

This question proved too difficult for almost all the students in the competition, compared to Question 8 in the 2024 competition. A reasonable number of students were able to identify the similar triangles, but finding the length of (say)* BE *was beyond most. Like Question 7, those who could get that far were able to finish the rest of the question without too much difficulty.