The University of Otago

Junior Mathematics Competition

2026 Questions for Part One

Department of Mathematics and Statistics
Te Tari Pākarau me te Tatauraka

University of Otago
ŌTĀKOU WHAKAIHU WAKA

These questions are for students in **Year 12**

Please ensure you have the right question booklet for your year level.

INSTRUCTIONS TO CANDIDATES

Please read the following Instructions carefully before you begin.

1. You have a maximum of **fifty minutes** to attempt any number of the questions in this booklet.
2. Each question is worth 1 mark. You will NOT be penalised for incorrect answers.
3. Your answer for each question should be recorded on the provided form. Answers not put onto this form will **not be marked**.
4. Please ensure your name, year, and school are recorded on the provided form.
5. Calculators and scrap paper are permitted, but textbooks are NOT allowed. Otherwise normal examination conditions apply.
6. Diagrams are not necessarily to scale nor accurate.

Note: There are **4** pages in this question booklet — this instruction page and **3** pages of questions.

DEFINITIONS

- A prime number has exactly two factors: itself and 1. The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.
- A number greater than 1 that is not a prime number is a composite number.

DO NOT TURN OVER UNTIL TOLD TO DO SO.

In physics, the half life of a decaying substance is the time it takes to reduce to half its initial quantity. A certain radioactive substance, Zn-71, has a half-life of 2.4 minutes.

(1) How much of a 100 g initial sample of Zn-71 would be left after 7.2 minutes?

(A) 25 g (B) 12.5 g (C) 10 g (D) 6.25 g

(2) How long would it take for a 48 g sample of Zn-71 to decay to 3 g?

(A) 4.8 minutes (B) 7.2 minutes (C) 9.6 minutes (D) 12 minutes

(3) An unknown quantity of the Zn-71 is placed in a box for 12 minutes. Afterwards, the remaining quantity of Zn-71 is found to weigh 5 g. How much Zn-71 was there when it was initially placed in the box?

(A) 100 g (B) 160 g (C) 320 g (D) 400 g

Jed is running on a treadmill. He starts by walking at 6 km/h. After 20 minutes, he increases his speed to 12 km/h and runs for another 40 minutes.

(4) What distance did Jed travel on the treadmill in total?

(A) 10 km (B) 12 km (C) 12.8 km (D) 14 km

(5) What was Jed's average speed in kilometres per hour (km/h)?

(A) 8 km/h (B) 9 km/h (C) 10 km/h (D) 11 km/h

Daniel is also running on a treadmill. He runs at a constant speed of 8 km/h, then slows down to a speed of 4 km/h. He runs for a total of 36 minutes, and completes a total distance of 4 km.

(6) For how long did Daniel run at 8 km/h?

(A) 12 minutes (B) 18 minutes (C) 24 minutes (D) 30 minutes

(7) Daniel wants to run 2026 kilometres in 2026. He goes to the gym four times a week, and runs the same number of kilometres each time. How many kilometres should he run each time he goes to the gym if he wants to achieve his goal? Round your answer to one decimal place.

(A) 6.2 km (B) 7.3 km (C) 8.4 km (D) 9.7 km

A cylindrical water tank is leaking water at a constant rate. In two minutes, the height of the water in the tank drops by 15 cm. In the same amount of time, the water that escaped exactly fills a 50 L container. (1 L = 1000 cm^3.)

(8) What is the rate of decrease of the height of the water in meters per second? Give your answer to the nearest 2 significant figures.

(A) 0.15 m/s (B) 0.0125 m/s (C) 0.013 m/s (D) 0.0013 m/s

(9) What is the rate of flow of the water out of the tank in cm^3/s? Give your answer to the nearest 2 significant figures.

(A) 430 cm^3/s (B) 420 cm^3/s (C) 416 cm^3/s (D) 410 cm^3/s

(10) What is the radius of the tank in cm? Give your answer to the nearest 2 significant figures.

(A) 33 cm (B) 32 cm (C) 28 cm (D) 25 cm

(11) The tank is 120 cm tall. What is the total capacity of the tank?

(A) 150 L (B) 200 L (C) 300 L (D) 400 L

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(12) What is the probability that the sum of two six-sided dice rolls is exactly 4?

(A) $1/12$ (B) $1/9$ (C) $1/6$ (D) $1/3$

(13) What is the probability that after three six-sided dice rolls neither a 2 nor a 6 is rolled?

(A) $64/216$ (B) $125/216$ (C) $152/216$ (D) $91/216$

(14) Three consecutive one digit integers are a perfect square, a perfect cube and a prime (not necessarily in that order). What is the product of these three integers?

(A) 60 (B) 210 (C) 336 (D) 504

(15) What is $\frac{2^8}{8^2}$ equal to?

(A) 2 (B) 4 (C) 16 (D) 64

(16) What is half of 4^{2026}?

(A) 2^{2026} (B) 4^{1013} (C) 2^{4051} (D) 2^{1013}

(17) What is the last digit of 3^{2026}?

(A) 1 (B) 3 (C) 7 (D) 9

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(18) How many different letter arrangements can be formed from the word SALT?

(A) 4 (B) 6 (C) 12 (D) 24

(19) How many different letter arrangements can be formed from the word PEPPER?

(A) 16 (B) 24 (C) 60 (D) 720

(20) How many more arrangements can be made from the word MILK than the word MILL?

(A) 2 (B) 6 (C) 12 (D) 24

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(21) If the side length of an equilateral triangle is r, what is its height?

(A) $\frac{r}{\sqrt{2}}$ (B) $\frac{\sqrt{3}r}{2}$ (C) $\sqrt{3}r$ (D) $\sqrt{2}r$

(22) What is the external angle of a regular octagon?

(A) 36° (B) 45° (C) 60° (D) 72°

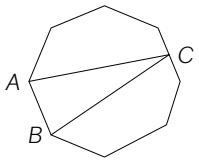

Figure 1

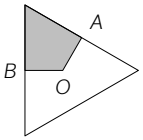

Figure 2

(23) In Figure 1 an isosceles triangle $\triangle ABC$ is constructed inside a regular octagon with a side length of 2 cm, such that A and B form one of the sides of the octagon and C is the middle of the side of the octagon opposite to AB. Which of the following is the closest to the area of $\triangle ABC$?

(A) 3.73 cm^2 (B) 4.23 cm^2 (C) 4.83 cm^2 (D) 5.14 cm^2

(24) In the equilateral triangle seen in Figure 2, A and B represent the midpoints of two sides, while O represents the triangle centre. If the side length of the triangle is 3 cm, which of the following is closest to the area of the shaded region within the triangle?

(A) 2.60 cm^2 (B) 1.06 cm^2 (C) 1.30 cm^2 (D) 3.62 cm^2

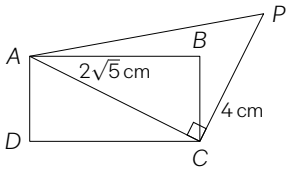

Figure 3

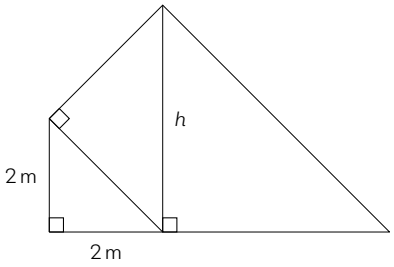

Figure 4

The next three questions refer to Figure 3.

(25) What is the distance AP?

(A) 4 cm (B) 6 cm (C) 8 cm (D) 36 cm

(26) The rectangle $ABCD$ has perimeter 12 cm. What is the area of the rectangle $ABCD$?

(A) 8 cm^2 (B) 12 cm^2 (C) 16 cm^2 (D) 24 cm^2

(27) What is the total area of the quadrilateral *APCD*?

(A) $4 + 4\sqrt{5}\,\text{cm}^2$ (B) $4 + 8\sqrt{5}\,\text{cm}^2$ (C) $16\,\text{cm}^2$ (D) $6 + 4\sqrt{5}\,\text{cm}^2$

A shape is made up of three similar right-angled triangles, as shown in Figure 4. The smallest triangle has two sides of side length 2 m.

(28) What is the area of the smallest triangle?

(A) $1\,\text{m}^2$ (B) $2\,\text{m}^2$ (C) $4\,\text{m}^2$ (D) $6\,\text{m}^2$

(29) What is the unknown height h?

(A) 4 m (B) 5 m (C) 8 m (D) 16 m

(30) What is the area of the shape?

(A) $12\,\text{m}^2$ (B) $14\,\text{m}^2$ (C) $12 + 12\sqrt{2}\,\text{m}^2$ (D) $24 + 20\sqrt{2}\,\text{m}^2$

Note: the volume of a sphere is $\frac{4}{3}\pi r^3$ where r is the sphere's radius. The volume of a cylinder is $\pi r^2 h$ where r is the cylinder's radius and h is its height. The volume of a cone is $\frac{1}{3}\pi r^2 h$ where r is the radius of the cone's base and h is its height.

A cone, a sphere and a cylinder all have the same radius, r. The cylinder is 3 cm tall.

(31) What is the radius of the cylinder in terms of its volume, V?

(A) $r = \sqrt{\frac{V}{3\pi}}$ (B) $r = \frac{V}{3\pi}$ (C) $r = \sqrt{\frac{3\pi}{V}}$ (D) $r = \frac{3\pi}{V}$

The sum of the volumes of the sphere and the cone is equal to the volume of the cylinder. Moreover, the sum of the heights of the sphere and the cone is equal to 2 times the height of the cylinder.

(32) What is the radius of the three shapes?

(A) 1 cm (B) 1.5 cm (C) 2 cm (D) 2.5 cm

(33) What is the volume of the cylinder?

(A) $5\pi\,\text{cm}^3$ (B) $6.25\pi\,\text{cm}^3$ (C) $6.75\pi\,\text{cm}^3$ (D) $7.5\pi\,\text{cm}^3$

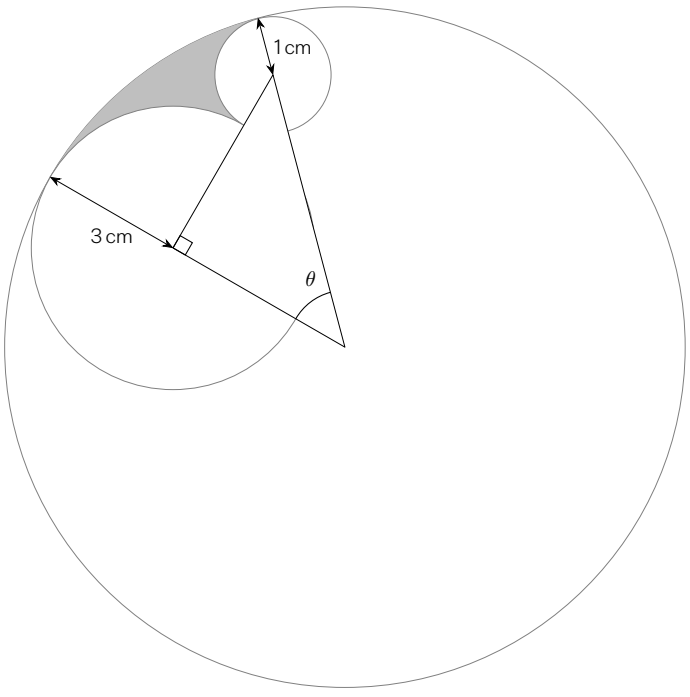

Figure 5

Each of the three circles in Figure 5 touch each other at one point. Their centres form a right angle triangle. The smallest circle has a radius of 1 cm, and the next smallest has a radius of 3 cm.

(34) What is the radius of the largest circle?

(A) 2 cm (B) 4 cm (C) 6 cm (D) 7 cm

(35) In degrees what is the angle θ, to one decimal place?

(A) 47.3° (B) 53.1° (C) 64.3° (D) 72.9°

(36) What is the perimeter of the shaded portion, to 1 decimal place?

(A) 10.5 cm (B) 12.8 cm (C) 13.6 cm (D) 14.1 cm

(37) What is the area of the shaded portion, to 1 decimal place?

(A) $1.5\,\text{cm}^2$ (B) $2.3\,\text{cm}^2$ (C) $2.4\,\text{cm}^2$ (D) $2.6\,\text{cm}^2$

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(38) What is the factorised form of $x^2 + x - 30$?

(A) $(x+5)(x-6)$ (B) $(x-3)(x+10)$ (C) $(x-5)(x+6)$ (D) $(x+3)(x-10)$

(39) Which of the following are the solutions to the equation $x^2 - 5x + 5 = 1$?

(A) $x = 2$ or $x = 3$ (B) $x = -2$ or $x = -3$ (C) $x = 4$ or $x = -1$ (D) $x = 4$ or $x = 1$

(40) How many solutions are there to the equation $(x^2 - 5x + 5)^{(x^2+x-30)} = 1$?

(A) 2 (B) 3 (C) 4 (D) 6

(END OF COMPETITION)