The University of Otago

Junior Mathematics Competition

2025 Questions

University of Otago
ŌTĀKOU WHAKAIHU WAKA

Department of Mathematics and Statistics   Te Tari Pākarau me te Tatauraka

INSTRUCTIONS TO CANDIDATES

You have a maximum of **fifty minutes** to answer **six** questions out of **eight**. The set of questions you answer is determined by your year level:

Question 1: 10 marks. Year 9 and below only.
Question 2: 10 marks. Year 10 and below only.
Question 3 to Question 6: 20 marks each. All students.
Question 7: 10 marks. Years 10 and 11 only.
Question 8: 10 marks. Year 11 only.

If you answer an incorrect question for your year level it will not be marked.

These questions are designed to test your ability to analyse a problem and express a solution clearly and accurately.

Please read the following Instructions carefully before you begin.

1. Do as much as you can. You are not expected to complete the entire paper. In the past full answers to three full (20 mark) questions have represented an excellent effort.
2. You must explain your reasoning as clearly as possible with a careful statement of the main points in the argument or the main steps in the calculation. Generally even a correct answer without any explanation will not receive more than half credit. Likewise clear and complete solutions to three full problems will generally gain more credit than sketchy work on four.
3. Credit will be given for partial solutions and evidence of a serious attempt to tackle a problem.
4. Textbooks are NOT allowed. Calculators may be used and students who do not have one may be disadvantaged. Otherwise normal examination conditions apply.
5. We recommend black or blue pens. Dark pencil is acceptable if you have nothing else. Do NOT use red or green pens, or light pencil that we cannot read.
6. We will penalise inappropriate rounding and incorrect or absent units.

Note: There are **four** pages in this question booklet: this instruction page and **three** pages of questions.

DEFINITION

A *prime number* has exactly two factors, 1 and itself. By this definition, 1 is not a prime number.

The first ten prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29.

DO NOT TURN OVER UNTIL TOLD TO DO SO.

Question 1: 10 marks (Years 9 and below only)

A 500 mL bottle of whole milk has the following nutritional values per 100 mL:

Nutrient	Amount per 100 mL
Protein	3.3 g
Fat	3.4 g
Carbohydrates	4.8 g

(a) Based on the values per 100 mL, calculate the total amount of protein, fat, and carbohydrates in the entire 500 mL bottle of milk.

(b) Given that protein and carbohydrates provide 4 kcal per gram and fat provides 9 kcal per gram, determine how many calories (in kcal) come from each nutrient in the 500 mL bottle.

(c) Calculate the total amount of calories (in kcal) from the three nutrients in the 500 mL bottle of milk. Determine what percentage of the total amount of calories comes from fat.

Question 2: 10 marks (Years 10 and below only)

Local honey producers process and package Manuka honey before selling it in the market. Three types of packaging, Large, Medium, and Small, are available with the following details:

Package Type	Weight (g per jar)	Production Cost ($ per jar)	Selling Price ($ per jar)
Large	400	35	45
Medium	300	28	36
Small	200	19	25

If each type of packaging has sold 120 kg of Manuka honey:

(a) Calculate the total number of jars sold for each package type.

(b) Which type of packaging contributes the most profit, and what is the total profit obtained for that type?

(c) Quinn wants to buy 600g of Manuka honey. What is the cheapest way for Quinn to purchase this amount?

Question 3: 20 marks (All Years)

If p, q, r are three consecutive prime numbers, q is a **balanced prime** if it is the mean of the two neighboring prime numbers p and r. For example, $5 = (3 + 7) \div 2$ and $157 = (151 + 163) \div 2$ are both balanced primes.

(a) Which of the following numbers is a balanced prime number?

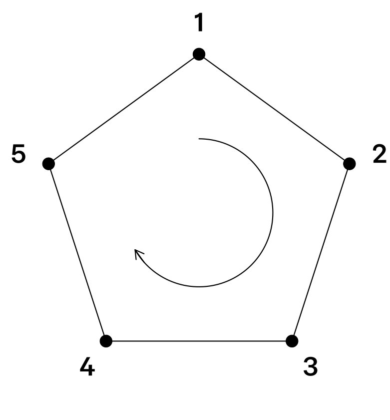
51, 53, 57, 59

(b) Check whether 97 is a balanced prime number.

(c) What is the smallest balanced prime number above 157?

Question 4: 20 marks (All Years)

Cayley is playing a game in a pentagon-shaped park where the corners are labelled 1, 2, 3, 4, and 5. If Cayley starts at one corner and walks along the edges in a clockwise direction, the number of steps he takes corresponds to the label of the corner where he started. This walking method is referred to as a "shift".



For example, if Cayley starts at corner 3, he would walk 3 steps: from corner $3 \to 4 \to 5 \to 1$. After the first "shift", he ends up at corner 1. Then, starting from corner 1, he would walk 1 step: from corner $1 \to 2$, completing the second "shift".

If Cayley starts at corner 2, answer the following questions:

(a) What is the corner number he reaches after 2 shifts?

(b) What is the corner number he reaches after 4 shifts?

(c) What is the corner number he reaches after 10 shifts?

To celebrate the arrival of the year 2025, Cayley makes a heroic move, starting at corner 1 and completing 2025 shifts in the pentagon-shaped park. Answer the following question:

(d) What is the corner number he reaches?

Question 5: 20 marks (All Years)

In a numerical transformation game, we define the numbers 0, 1, 2, ..., 50 as "old numbers". To transform an old number, the following steps are used:

(1) Square the old number.

(2) Divide the result by 100.

(3) The remainder is the "new number".

Example: For the old number 39,

$$39^2 = 1521, \quad 1521 \div 100 = 15 \text{ with remainder } 21$$

So, the new number for 39 is 21.

(a) Apply this rule to transform 26 into its corresponding new number.

(b) Given a new number 36, find its corresponding old number(s).

(c) After transformation, many numbers decrease in value. Jacob claims, "No new number is equal to its old number". Do you agree with Jacob? If not, find all old numbers for which the new number remains the same.

(d) Identify the old number that decreases the most after transformation and show your working.

Question 6: 20 marks (All Years)

A vending machine offers three types of snacks: chips, chocolate, and cookies. Customers can select snacks based on their preferences.

(a) In how many ways can a customer choose exactly two different snacks from the three available options?

Later, the vending machine adds a fourth snack option: juice, providing more choices.

(b) How many ways can a customer now select exactly two different snacks from the four available options?

(c) If a customer can choose any two snacks, but they don't have to be different (i.e., they can pick two of the same snack), how many different selections are possible?

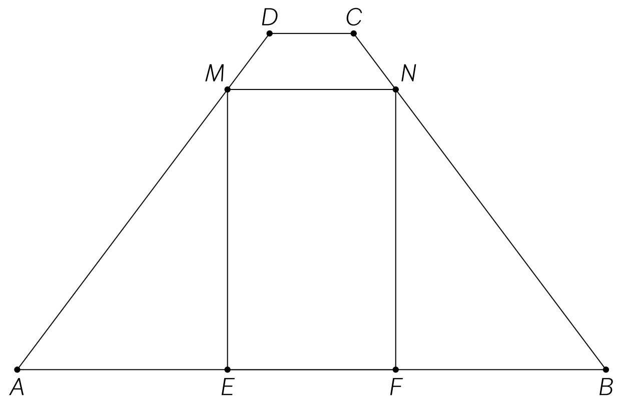
(d) If the vending machine limits each customer to a maximum of three snacks per purchase (they don't have to be different), how many different ways can a customer select their snacks from the four options?

Question 7: 10 marks (Years 10 and 11 only)

In the trapezium $ABCD$, $AB \parallel CD$, where:

$$AB = 7, \quad CD = 1, \quad AD = BC = 5$$

Points M and N are on AD and BC, respectively. We have $MN \parallel AB$, $ME \perp AB$ and $NF \perp AB$, with E and F being the respective foot points.



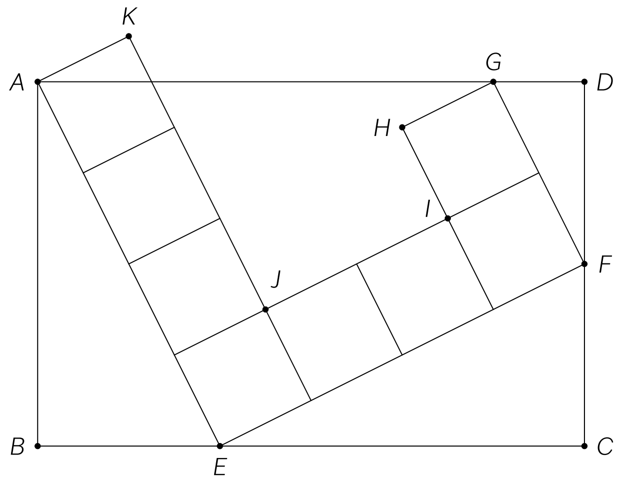
(a) Calculate the area of the trapezium $ABCD$.

(b) Express the area of the quadrilateral $MEFN$ in terms of x, where $AE = x$.

(c) If points M and N can move along AD and BC, respectively, while always ensuring $MN \parallel AB$, determine whether the quadrilateral $MEFN$ can be a square. If yes, find its area; if not, explain why.

Question 8: 10 marks (Year 11 only)

In the given diagram, we have a rectangle $ABCD$ and a polygon $AEFGHIJK$ composed of 8 identical small squares, each with an area of 1 unit. Points E, F and G are on the perimeter of the rectangle $ABCD$.



Determine the perimeter of the rectangle $ABCD$.

(END OF COMPETITION)