2013

Junior Mathematics Competition

UNIVERSITY of OTAGO
Te Whare Wānanga o Otāgo
NEW ZEALAND

Questions

TIME ALLOWED: ONE HOUR

Only Year 9 candidates may attempt QUESTION ONE

ALL candidates may attempt QUESTIONS TWO to FIVE

These questions are designed to test ability to analyse a problem and to express a solution clearly and accurately.

Please read the following instructions carefully before you begin:

(1) Do as much as you can. You are not expected to complete the entire paper. In the past, full answers to three questions have represented an excellent effort.

(2) You must explain your reasoning as clearly as possible, with a careful statement of the main points in the argument or the main steps in the calculation. Generally, even a correct answer without any explanation will not receive more than half credit. Likewise, clear and complete solutions to two problems will generally gain more credit than sketchy work on four.

(3) Credit will be given for partial solutions and evidence of a serious attempt to tackle a problem.

(4) Textbooks are NOT allowed. Calculators may be used and students who do not have one may be disadvantaged. Otherwise normal examination conditions apply.

(5) Diagrams are a guide only and are not necessarily drawn to scale.

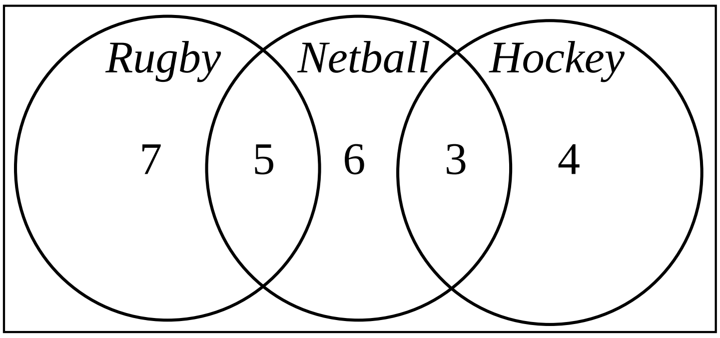
(6) We recommend black or blue pens. Dark pencil is acceptable if you have nothing else. Do NOT use red or green pens, or light pencil that we cannot read.

DO NOT TURN THIS OVER UNTIL YOU ARE TOLD.

Question 1 (Year 9 and below only)

Venn Diagrams are useful for solving certain problems.

a) The diagram shows a Venn diagram for a class of 30 students and three of the sports they play. For example, the circle labelled 'RUGBY' shows that 12 of the pupils play rugby. Of those 12, 5 also play netball. Nobody in the class plays both rugby and hockey.



(i) How many people in the class play netball?

(ii) How many people in the class don't play any of these three sports?

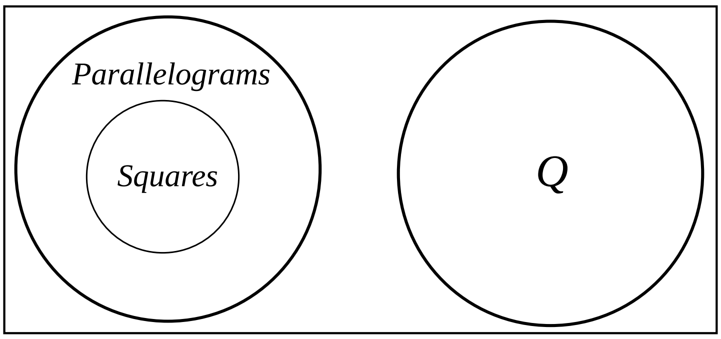
b) On the answer booklet (not in the questions) draw up your own Venn Diagram with three circles labelled 'COLD BLOODED ANIMALS', 'WARM BLOODED ANIMALS' and 'FISHES' to illustrate the statement 'All fishes are cold blooded animals. Some, but not all, cold blooded animals are fishes'. Note: Not all fishes are cold blooded, but for the purpose of the question you may regard them as cold blooded.

c) On the answer booklet (not in these questions) draw another Venn Diagram for a different class, then answer the question. In this class there are 32 students. 18 play rugby, of whom 8 also play netball and 4 play hockey. 12 play netball, of whom 8 also play rugby (already mentioned) and 4 also play hockey. 8 play hockey, and of those some also play rugby or netball (already mentioned), but 2 students play all three sports.

Question: How many students play none of these sports?

d) The following Venn Diagram shows PARALLELOGRAMS, SQUARES, and an unknown quadrilateral called 'Q'. Copy the diagram into your Answer Booklet (do NOT answer on the question sheet), and on <u>your</u> diagram add a circle in the correct place that shows RECTANGLES.

Note: Perfect circles are NOT needed.



TURN OVER

Question 2 (All years)

The year 2013 has the digits '0', '1', '2', and '3' repeated only once in it.

a) Find a 4 digit number (where the first digit is **not** '0') different to 2013 which uses each of '0', '1', '2', and '3' once and only once.

b) Find and list all 4 digit numbers starting with '3' which use '0', '1', '2', and '3' once and only once.

c) How many 4 digit numbers (where the first digit is **not** '0') are there that use '0', '1', '2', and '3' once and only once?

d) How many 4 digit numbers are there which start with 4, have no repeating digits, and do not use any of '0', '1', '2', and '3' at all?

e) How many 4 digit numbers are there altogether that have no repeating digits and do not use any of '0', '1', '2', and '3' at all?

Question 3 (All Years)

King Arthur was happiest when his Knights of the Round Table sat around the table in a special way. If there were n knights present, he would give them numbers 1, 2, 3, . . . n, and then ask them to sit so that each adjacent pair of knights had numbers that summed to the same value as the adjacent pair of knights sitting diagonally opposite them.

The picture in Figure 1 shows the arrangement one day when six knights were present.

Here 1 + 6 = 4 + 3, 6 + 2 = 3 + 5, and 2 + 4 = 5 + 1.

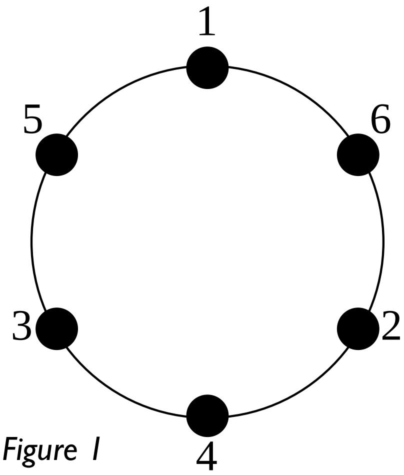


Figure 1

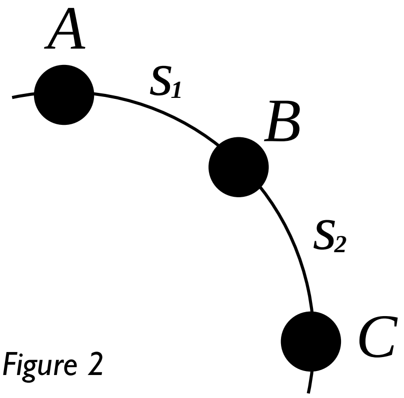


Figure 2

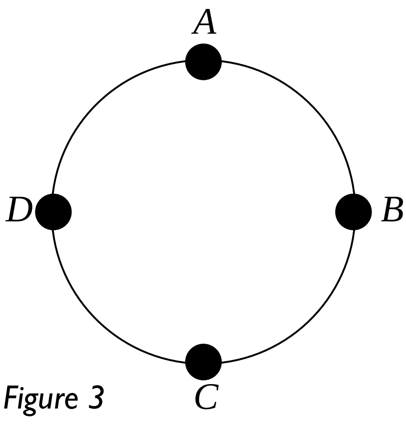


Figure 3

a) Find another arrangement of the six knights that satisfies Arthur's wishes. (Do not use a simple reflection of the one already shown.) You may either draw a circle showing the six numbers, or list the numbers in sequence.

b) Why must the number of knights n be even for this to work?

c) Can Arthur's wishes be satisfied with just four knights? (Answer by trying out the possible seating arrangements with 1 at the top. Answers stating 'yes' or 'no' without working will gain no credit.)

d) Consider as shown in Figure 2 the three adjacent knights with numbers A, B and C, and sums s_1 and s_2. (For example, s_1 = A + B.)

Show that C - A = s_2 - s_1.

e) Suppose four knights are seated as shown in Figure 3. Use the result in part (d) to show that Arthur's wishes would need A = C.

(This is obviously not possible, hence proving that there is no solution for n = 4.)

f) Show that there is no solution when n is a multiple of 4.

TURN OVER

Question 4 (All years)

A racing track consists of two 100 m straights and two semicircles of 100 m each at each end (see diagram).

For the purpose of this question regard it as a line with no width.

a) How many full laps of the track are needed for the 10 000 m race?

b) Show that the diameter of the circular sections (i.e. from A to C) must be 64 m, to the nearest metre.

If you cannot work this value out, then use it (if necessary) in the rest of the question. (A, B, C, and D are the corners of the rectangular part of the field inside the track.)

c) Find the total area of the enclosed track. Give your answer in hectares. (A hectare is 100m by 100m).

d) How far is it from corner to corner i.e. diagonally from A to D? Give your answer to the nearest metre.

Question 5 (All students)

In this question, give probability answers in decimal form to six decimal places if necessary. For example, if you think an answer is ½, for full marks give the answer as 0.5. (There is no need to give this answer to 6 decimal places.)

A fair chocolate wheel at a school fund-raiser has 20 numbers from 1 to 20. Each number has an equal chance of occurring when the wheel is spun.

a) On one draw, the wheel is spun once. You have bought one ticket; the ticket has one number on it in the range 1 to 20, and that the ticket wins if the wheel stops on that number.
What is the probability of your winning?

b) Later the wheel is spun three times. You have one ticket that is valid each time the wheel is spun, i.e. it can win on each and every spin. What is the probability of

(i) zero wins,

(ii) exactly one win,

(iii) exactly three wins?

c) At the end of the day, the wheel is spun eight times. You have one ticket, valid each time the wheel is spun. On draws one, three, five, and seven, the prize is a large soft toy. On the other four draws, the prize is a small soft toy.

(i) In how many ways can you win exactly one large toy and exactly one small toy?

(ii) What is the probability of your winning exactly one large toy and exactly one small toy?

END OF COMPETITION