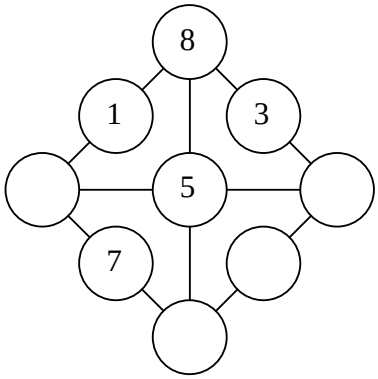
Question 1 (Year 9 or below only)



The diagram at the right show a Magic Diamond. Each circle is supposed to contain one of the numbers from 1 to 9, with no repeats. However, some numbers are missing from the diagram.

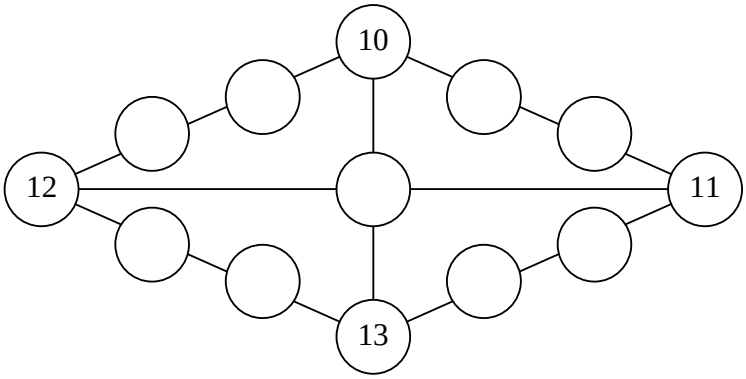
In a Magic Diamond the three numbers along each line always add up to the same total.

(a) Write down in order from smallest to largest the four missing numbers from the Magic Diamond.

(b) What is the total which the three numbers along each line always add up to?

(c) Carefully copy the Magic Diamond above into your Answer Booklet and write in the missing numbers so that the three numbers along each line always add up to the total you wrote down in (b).

Marlon tries to draw a larger Magic Diamond. His Magic Diamond contains all the numbers from 1 to 13. Marlon's Magic Diamond is shown below. Once again some of the numbers are missing. The numbers (sometimes three of them and sometimes four) along each line always add up to the same total.



(d) What is the total which the numbers along each line always add up to?

(e) Carefully copy Marlon's Magic Diamond into your Answer Booklet and write in the missing numbers so that the numbers along each line always add up to the total you wrote down in (d).

Question 2 (All Students)

Sally stores mathematical expressions in her calculator. She uses special function buttons $\boxed{f_1}$ $\boxed{f_2}$ etc.
to store expressions which she wants to use later.

One day her little brother Robert is looking at her calculator and he asks Sally about the special function buttons. Sally shows him how they are used (although she doesn't use any fractions at all because Robert doesn't really understand fractions yet).

Sally tells Robert that the expression she stored using f_1 is $4x^2$. This means that if Robert enters a number into this expression, the number will be squared and multiplied by 4.

(a) Write down the result when Robert enters the number 3 into the expression Sally stored using f_1.

(b) When Robert presses **INV** f_1 the calculator undoes the expression stored using f_1. This means that if Robert uses **INV** f_1 for the result he obtained in part (a) then the answer would be 3. Write down the result when Robert uses **INV** f_1 for the number 64.

Sally doesn't tell Robert what expression she stored using f_2, but she does tell him that if he enters 2 and uses f_2 the result will be 12, while if he enters 3 and uses f_2 the result will be 36.

(c) Find a possible result when the number 4 is entered using the stored result f_2. Explain what expression has been stored using f_2 to give your result.

(d) Is the result you found in (c) the only possible result? If it is, explain why. If it isn't, write down another possible result when the number 4 is entered using f_2 and also give the expression you used to obtain your new result.

Question 3 (All Students)

For this question, assume that the planet Earth is a perfect sphere. During November 2006, residents of Dunedin were able to take helicopter rides to look at several icebergs which were floating off the coast.

(a) One day, when conditions were clear and sharp, viewers were able to see an iceberg on the horizon from the top of a local hill, Flagstaff, which is 660 m above sea level. If the distance in a direct line from Flagstaff to the iceberg was estimated to be 90 km, use this information to show that the radius of the Earth from the centre of the planet to sea level is between 6000 km and 7000 km.

(b) If the iceberg broke off from Antarctica and then travelled in a straight line (relative to the surface) covering one twelfth $\left(1/12\right)$ of the circumference of the Earth before it was seen near Dunedin, find the distance it had travelled before it was seen. (Note that if you do not have a calculator then you may use the values 6000 km for the radius of the Earth and $\pi = 3.14$.)

Question 4 (All Students)

The 2007 Kakanui Bowls tournament was a knock-out competition. This means that if there were exactly 32 competitors then the first round of games consisted of each person playing against someone else, with the loser of each game being eliminated. The 16 competitors left then played in the next round, with the loser of each of these eight games being eliminated. This arrangement continued until only two players were left. The winner of this final game won the tournament.

However if there were extra competitors, for example an extra three (making 35 altogether), then six competitors first of all had to play a preliminary game (three games would be needed) to select the three competitors who would go through to be part of the last 32.

(a) How many games were there altogether if there were 35 competitors?

(b) When the entries closed it was discovered that there were 2007 competitors. The knock-out method described above was employed for this larger number of competitors. Boris, the eventual winner of the tournament, was one of those who first had to play a preliminary game. How many games did Boris have to play?

(c) How many games were there altogether for this number (2007) of competitors?

(d) If there are n full rounds of games plus m preliminary games, write down an expression for the total number of games which must be played in the tournament.

Question 5 (All Students)

(a) Paul has a sheet of graph paper which has 6 squares along each side. He draws squares of side lengths 1 through to 5 on this paper, along the lines. Show that no matter how he does this there will be a square on the paper which is part of at least three of the squares that he has drawn.

(b) Paul now takes another sheet of graph paper which has 40 squares along each side. On this sheet he draws squares of side lengths 1 through to 39 on this paper, along the lines. Show that no matter how he does this there will be a square on the paper which is part of at least 20 of the squares that he has drawn.

(c) For the situation in (b), is it possible for there to be exactly one square of this sort, i.e. exactly one square which is part of at least 20 of the squares that he has drawn?