Model-based Priors for Network Tomography

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Network Tomography

Goal: estimate origin-destination rates of traffic flow over a network.

Data: traffic counts at various network locations.

Applications: road networks, electronic communication networks.

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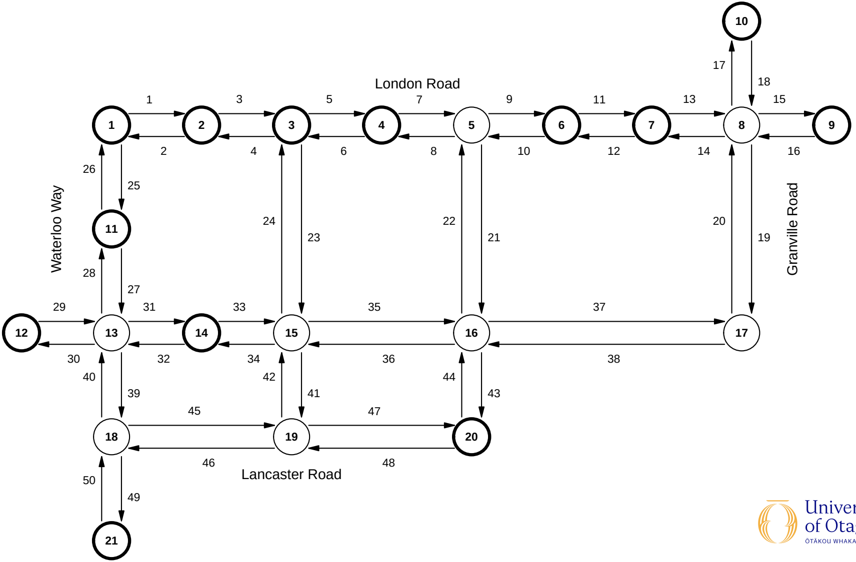
Illustrative Example

Leicester, U.K.



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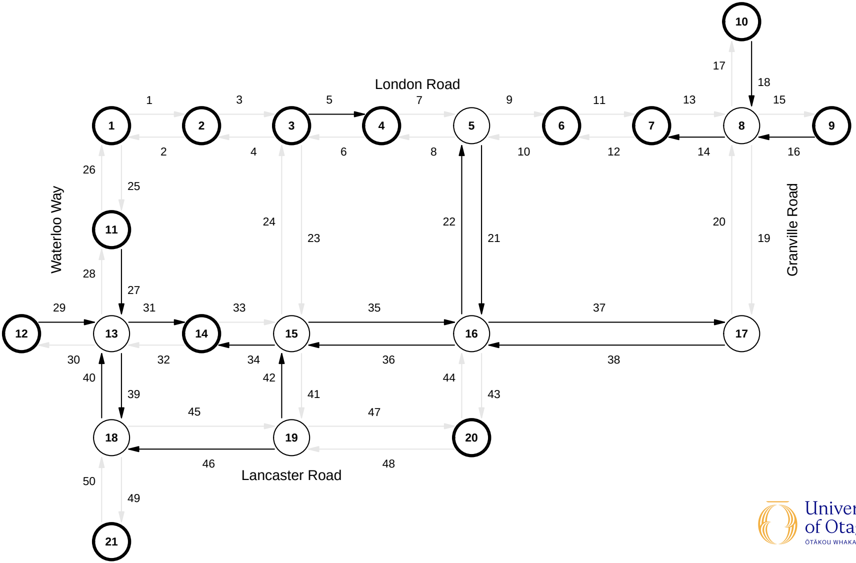

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More Questions Than Answers

Number of origin/destination nodes		13
Number of origin-destination pairs	$13 \times 12 =$	156
Number of traffic count sites		18

$d = 156$ parameters $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_d)^{\mathsf{T}}$ to estimate.
$n = 18T$ traffic count data points $\boldsymbol{y}$ available, over T periods.
Usually prior information on origin-destination rates $\boldsymbol{\theta}$ is available.

Routed

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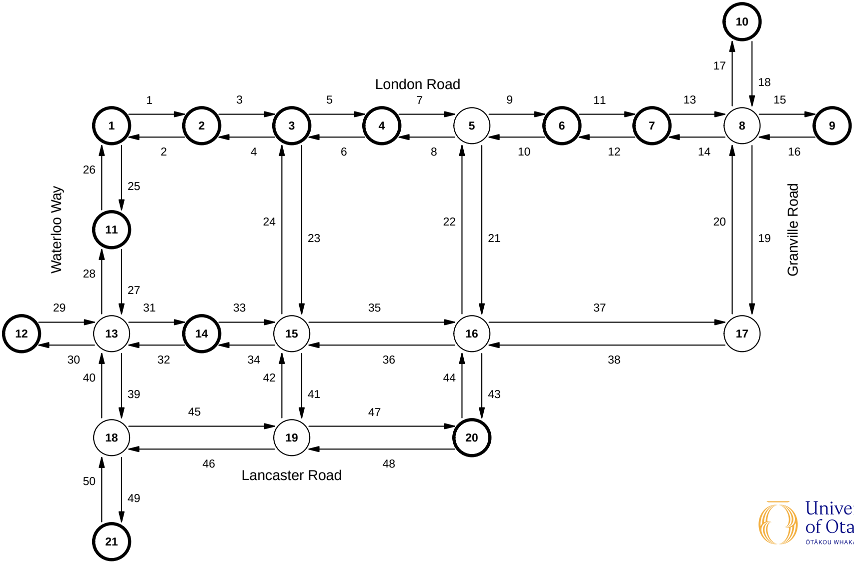

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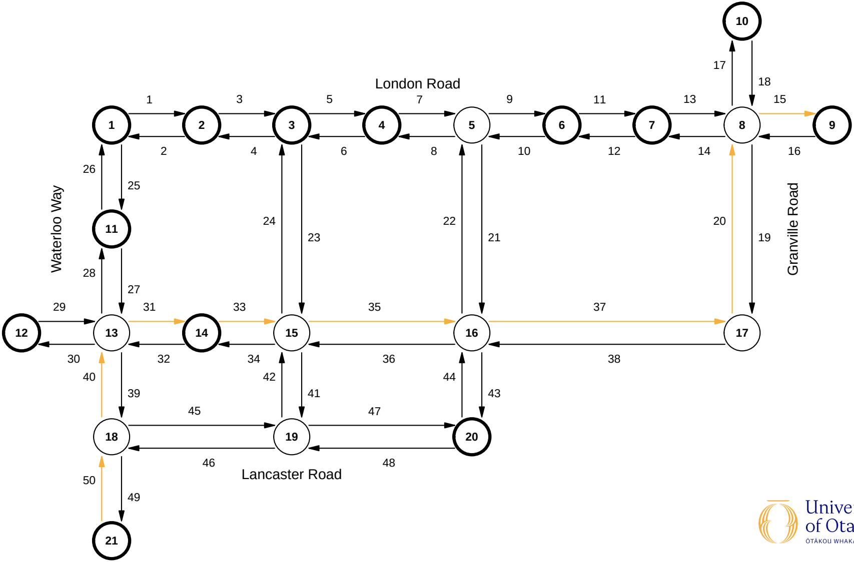

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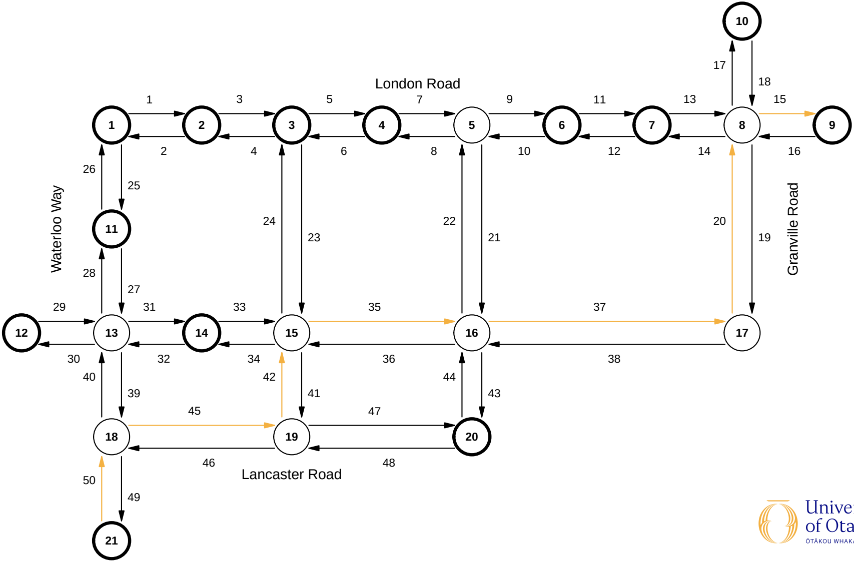

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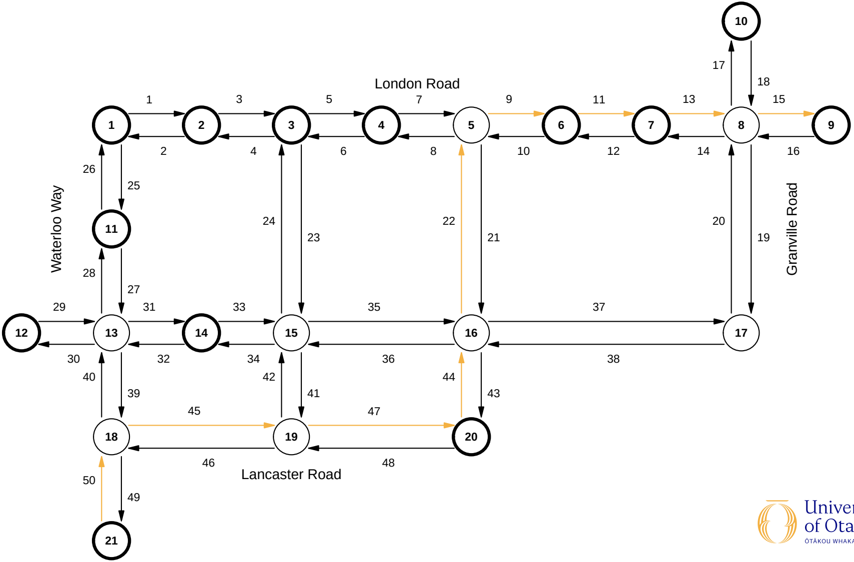

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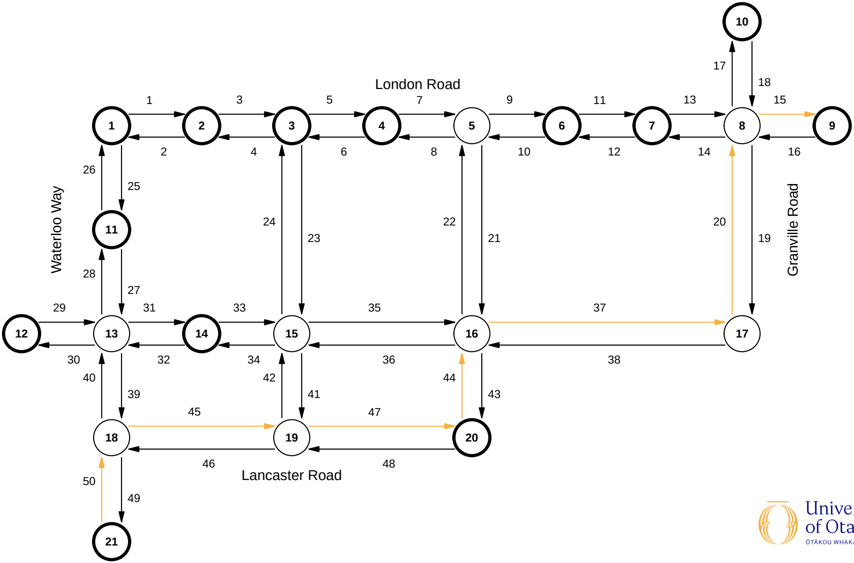

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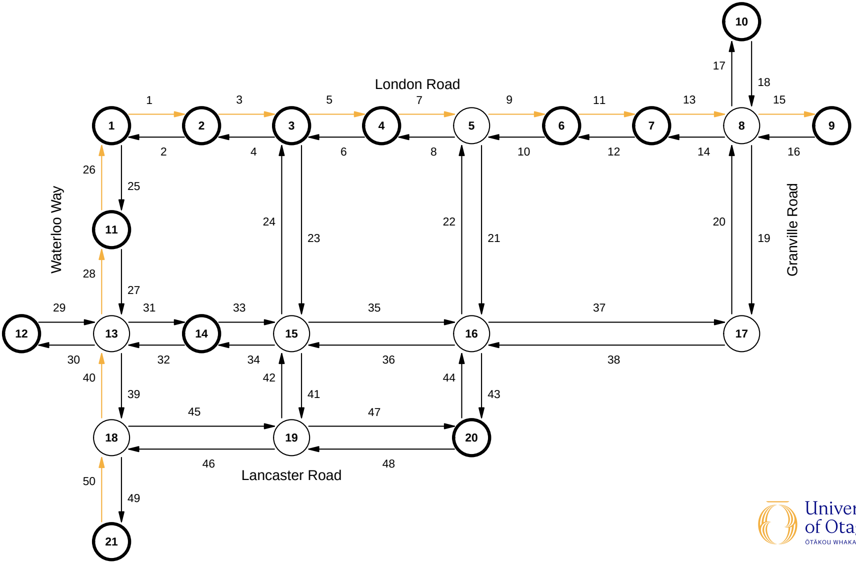

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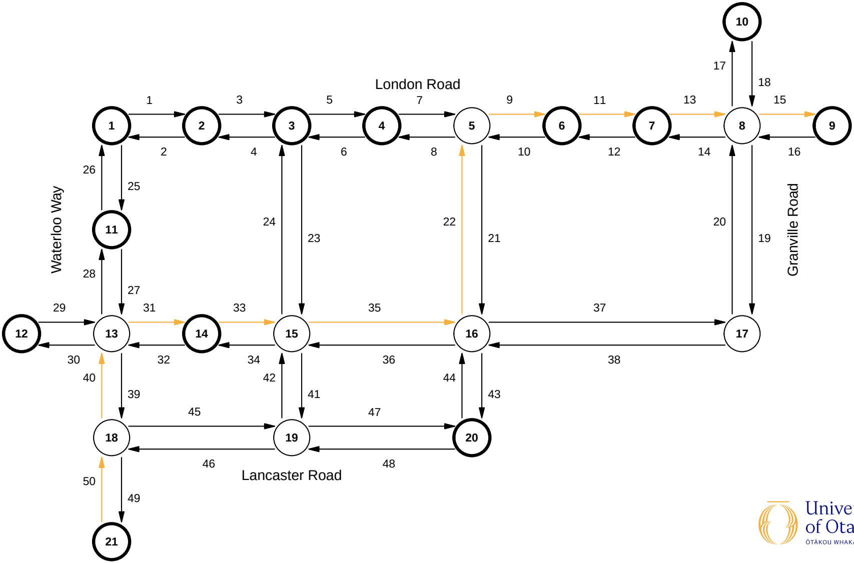

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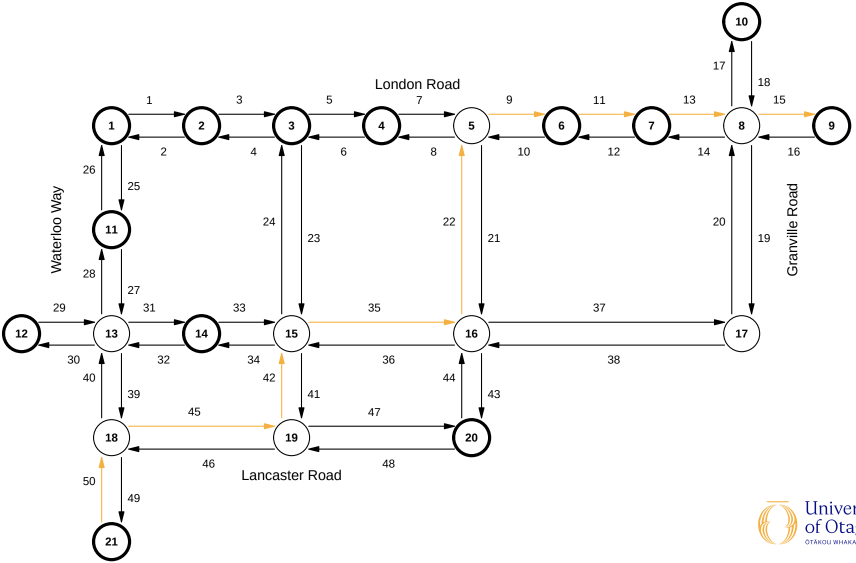

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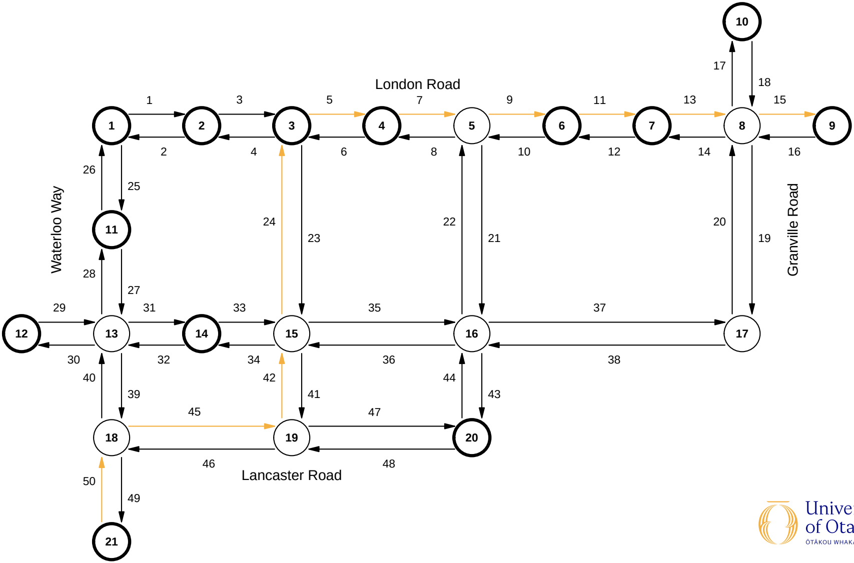

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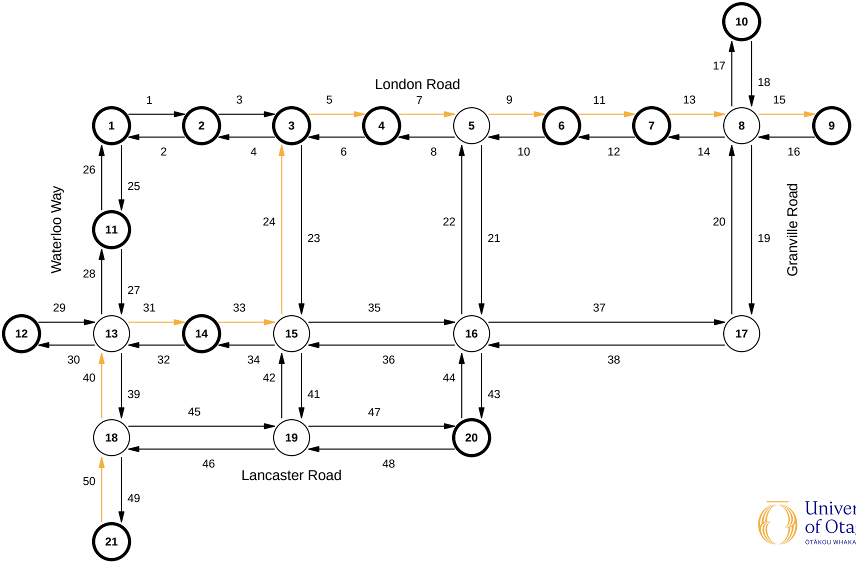

Route Choice probabilities

Nuisance Parameters

Number of origin-destination pairs	156
Number of plausible routes (paths)	355
Number of nuisance parameters	199

$r - d = 355 - 156 = 199$ free route choice probabilities

Limited explicit prior information.

Route choice will depend on traffic congestion.

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Modelling Route Choice Probability

Traffic Equilibrium Models

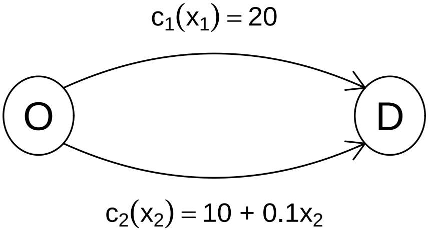
Long history of modelling route choice in transportation research

Deterministic equilibrium models, based on game theory, dominated historically.

- Premise is that route choices depend on travel times ...
- ... which depend on levels of congestion ...
- ... which depend on route choices.

Stochastic User Equilibrium (Daganzo & Sheffi, 1977)

Illustrative Example



Travel demand $\theta = 100$.

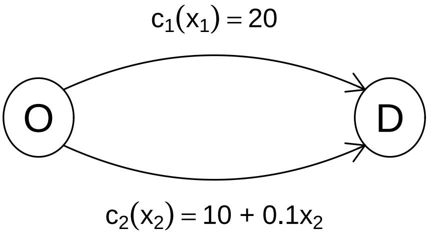
Stochastic User Equilibrium (SUE) is $x_1 = 16.3$, $x_2 = 83.7$, since

$$\text{P}(\text{route } 1) = \frac{e^{-c_1(16.3)}}{e^{-c_1(16.3)} + e^{-c_2(83.7)}} = \frac{e^{-20}}{e^{-20} + e^{-18.4}} = 0.163$$

Daganzo, C. F., & Sheffi, Y. (1977). On stochastic models of traffic assignment. *Transportation science*, **11(3)**, 253–274.

Stochastic User Equilibrium

Incorporating Cost Sensitivity



Introduce cost sensitivity parameter ω:

$$\mathrm{P}(\text{route } 1) = \frac{e^{-\omega c_1}}{e^{-\omega c_1} + e^{-\omega c_2}}$$

Stochastic User Equilibrium

Applied to Mean Traffic Flows

Stochastic User Equilibrium (SUE) route probabilities $\boldsymbol{p}$ satisfy fixed point equation:

$$\boldsymbol{p} = \boldsymbol{q}(A^{\mathsf{T}}\boldsymbol{c}\,(A\,\mathrm{diag}(\theta)\boldsymbol{p}))$$

- A is link-path incidence matrix;
- $\boldsymbol{c}$ is (vector-valued) link cost (travel-time) function;
- $\boldsymbol{q}$ is probability model, e.g. logit.
- Recall θ is mean vector for origin-destination traffic volumes.

Critical idea: employ SUE model to implicitly define prior for route choice probabilities as function of θ.

MCMC for Bayesian Inference

Define $\boldsymbol{x}$ to be vector of latent route traffic volumes.

Then $\boldsymbol{y} = A\boldsymbol{x}$.

Model origin-destination volumes as $\mathrm{Pois}(\boldsymbol{\theta})$, so $\boldsymbol{x} \sim \mathrm{Pois}(\mathrm{diag}(\boldsymbol{\theta}_{ext})\boldsymbol{p})$.

Outline of algorithm:

1. Sample latent $\boldsymbol{x}$ given $\boldsymbol{y}$, $\boldsymbol{\theta}$.
   - Possible by dynamic Markov basis fibre sampling (H. et al., 2021).
2. Sample $\boldsymbol{\theta}$ given $\boldsymbol{x}$.
   - Requires evaluation of acceptance probability $\propto [\boldsymbol{x} \mid \boldsymbol{\theta}][\boldsymbol{\theta}]$.
   - Requires evaluation of SUE to compute $\boldsymbol{p}(\boldsymbol{\theta})$

Hazelton, M., McVeagh, M., & Van Brunt, B. (2021). Geometrically aware dynamic Markov bases for statistical linear inverse problems. *Biometrika*, **108(3)**, 609–626.

SUE Emulation

Need to evaluate SUE probabilities at each iteration of sampler.

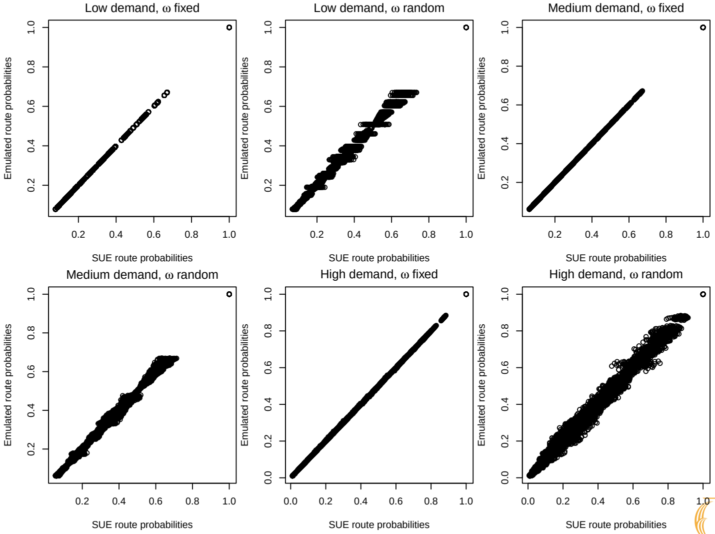
Solving fixed point equation for $\boldsymbol{p}(\theta)$ is computationally intensive.

Employ regression-based emulator, working on scale of travel-times.

- Linearizes SUE costs as function of θ.
- Uses training set with θ sampled from prior.

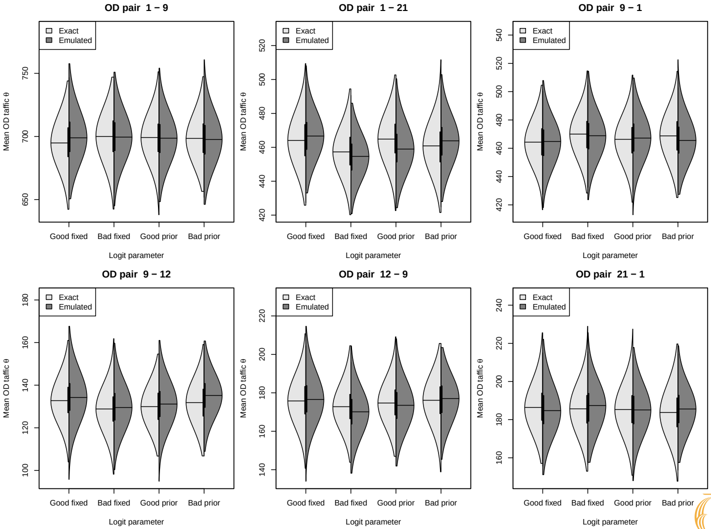
Numerical testing

Exact SUE Versus Emulated



Numerical testing

Impact of Emulation on Demand Estimation



Conclusions

- Models from transportation science can provide useful prior information for network tomography.
- Implementation using emulation necessary to control computational expense.
- Linear inverse nature of problem makes inference challenging:
  - How much information do data provide about sensitivity parameter(s)?
  - Reliability of travel time (cost) function specification?
  - How to conduct useful assessment of model fit?

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Funding

Thanks to ...

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To Learn More ...

Hazelton, M. L., & Najim, L. (2024). Using traffic assignment models to assist Bayesian inference for origin-destination matrices. *Transportation Research Part B: Methodological*, **186**, 103019.

R packages

`https://github.com/MartinLHazelton/Leicester`

`https://github.com/MartinLHazelton/LinInvCount`

`https://github.com/MartinLHazelton/transportation`

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