

# Stochastic Processes of Variable Fractional Order: Dirichlet Forms and Feller Processes

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We are interested in various forms of variable-order non-local operators which give rise to stochastic processes. There are several possibilities to introduce such operators; the most popular is, probably, the *stable-like* generator  $-(-\Delta)^{\alpha(x)}$  considered by R. Bass et al. Our approach is to consider (generalized) stable-like Dirichlet forms of the following type

$$\mathcal{E}(u, v) = \iint (u(y) - u(x))(v(y) - v(x)) \frac{dx dy}{|x - y|^{n+\alpha(x,y)}}$$

where  $\alpha(x, y)$  is a function on  $\mathbb{R}^n \times \mathbb{R}^n$  and values in  $\mathbb{R}$ . If  $\alpha(x, y) = \alpha(x) \in (0, 2]$  we are, essentially, in the stable-like setting. In this talk we address the following points

- *Existence of an associated stochastic process.* For this we use the theory of Dirichlet forms and give necessary and sufficient criteria in terms of  $\alpha(x, y)$ .
- *Structure of the infinitesimal generator and its symbol.* In general, it is very hard to calculate the explicit form of the generator of a Dirichlet form. In our case it turns out that under certain integrability assumptions on the kernel  $|x - y|^{-\alpha(x,y)-n}$  the generator is a pseudo-differential operator with negative definite symbol  $p(x, \xi)$ . From the symbol it is easily possible to read off the complete semimartingale characteristics of the associated jump-diffusion (in the sense of Jacod-Shiryaev).
- *Probability estimates.* Using the symbol we obtain estimates for the first passage times  $\sigma_r^x$  of the associated stochastic process from a ball  $B_r(x)$ . Typically, such estimates look like

$$c \left/ \sup_{|x-y| \leq r} \sup_{|z|=1} \operatorname{Re} p(y, z/r) \right. \leq \mathbb{E}^x \sigma_r^x \leq C \left/ \inf_{|x-y| \leq r} \sup_{|z|=1} \operatorname{Re} p(y, z/r) \right.$$

- *Existence of a good version of the process.* A major problem in the theory of Dirichlet forms is the existence of capacity-zero exceptional sets. Combining the above probability estimates with techniques due to Bass and Levin, Song and Vondracek, and Bass and Kaßmann we are able to show that certain  $\mathcal{E}$ -harmonic functions are (Hölder-)continuous. This can then be used to construct a Feller version of the Resolvent and the associated semigroup. This allows us to pick a ‘good’, i.e. Feller-version of the process without any exceptional sets. Using perturbation techniques one sees that our arguments are still valid if we change the kernel  $|x - y|^{-n-\alpha(x,y)}$  whenever  $|x - y| > \epsilon$ , i.e. off the diagonal.

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