Fractional Calculus Models in Finance

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Fractional partial differential equations are useful to model anomalous diffusion, where a plume of particles spreads differently than the classical diffusion equation predicts, with possible skewness and heavy power-law tailing. The same models are also useful in finance, where price jumps replace the particle jumps in the diffusion model. The corresponding stochastic models are called continuous time random walks (CTRWs), which are random walks that also incorporate a random waiting time between jumps. In finance, the waiting times measure delay between transactions. These two random variables (price change and waiting time) are typically not independent. For these CTRW models, we can now compute the limiting stochastic process (just like Brownian motion is the limit of a simple random walk), even in the case of heavy tailed (power-law) price jumps and/or waiting times. The transition density functions for this limit process solve fractional partial differential equations. In some cases, these equations can be explicitly solved to yield descriptions of long-term price changes, based on a high-resolution model of individual trades that may include statistical dependence between waiting times between trades and the subsequent price changes. In the heavy tailed case, this involves operator stable space-time random vectors that generalize the familiar stable models. In this talk, we will review the fundamental theory and present two applications: High-resolution LIFFE bond futures data from September 1997; and GE stock data from October 1999.