Diffusion Regimes in Brownian Motion Induced by the Basset History Force

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In this invited lecture I deal with a variant of the classical hydrodynamic approach to the Brownian motion making also use of the Fractional Calculus and its "Queen Function", the Mittag-Leffler function. The velocity autocorrelation and the displacement variance of a Brownian particle moving in an incompressible viscous fluid are known to be the fundamental quantities that characterize the evolution in time of the related stochastic process. Here they are calculated taking into account the effects of added mass and both Stokes and Basset hydrodynamic forces that describe the friction effects, respectively in the steady state and in the transient state of the motion. The explicit expressions of these quantities versus time are computed and compared with the respective ones for the classical Brownian motion. The effect of added mass is only to modify the time scale,that is the characteristic relaxation time induced by the Stokes force. The effect of the Basset force, which is of hereditary type namely history-dependent, is to perturb the white noise of the random force and change the decay character of the velocity autocorrelation function from pure exponential to power law because of the presence of functions of Mittag-Leffler type of order 1/2. Furthermore, the displacement variance is shown to exhibit, for sufficiently long times, the linear behaviour which is typical of normal diffusion, with the same diffusion coefficient of the classical case. However, due to the Mittag-Leffler functions, the Basset history force induces a very long retarding effect in the establishing of the linear behaviour allowing for a regime which could appear as a manifestation of anomalous diffusion of the slow type (sub-diffusion). This lecture is partly based on Author's works carried out with the collaboration of F. Tampieri (CNR, Bologna). Inspiring discussions with R. Gorenflo, G. Pagnini, E. Scalas and A. Vivoli are much appreciated.

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