From Diffusion to Anomalous Diffusion: A Century after Einstein's Brownian Motion

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The description of Brownian motion in Einstein's 1905 work relies on the assumption of the existence of a time-interval , such that the particle's motion during different -intervals is independent. The coarse-grained version of this motion leads to the known diffusion equation. However, in many cases this assumption is violated. An example is dispersive transport in disordered systems which stems from a broad distribution of waiting times which may have a diverging mean. This ill-defined mean waiting time results in subdiffusion. In this contribution we derive, within a unified scheme, two equivalent forms of kinetic equations for subdiffusive behavior. For power-law waiting-time distributions, the equations reduce to the "normal" form of a fractional Fokker-Planck equation with a fractional derivative replacing the first-order time-derivative, or to a "modified" form. For waiting time distributions which are not pure power laws one or the other form of the kinetic equation are shown to be advantageous, depending on whether the process slows down or accelerates in the course of time. Relationships to continuous time random walks and Levy processes will be discussed.