Fractional Reaction-Diffusion Equations and Pattern Formation

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Fractional reaction-diffusion equations provide useful models of diffusing and reacting species when the diffusion is anomalous sub-diffusion. These equations can be derived in the asymptotic long time limit from a mesoscopic description in terms of Continuous Time Random Walks with sources and sinks when the waiting time probability density corresponds to a heavy tailed distribution. The generic fractional reaction-diffusion system has fractional order temporal derivatives operating on the spatial Laplacian and reaction terms determined by the law of mass action. We have investigated Turing instability induced pattern formation in this generic model and in a related model with fractional order temporal derivatives operating on both the spatial Laplacian and the reaction terms. We have also carried out numerical simulations of these model systems. The linear Turing instability analysis provides a reliable indicator of both the onset and the nature of the patterns that form. Anomalous diffusion with reactions can produce complex spatio-temporal patterns that do not occur in standard reaction-diffusion models.