# New Zealand Mathematical Society Colloquium 2018

University of Otago
Dunedin, 4 – 6 December 2018

Conference booklet
*Version 10*
*03/12/2018*

# 1 Welcome

Welcome to the 2018 New Zealand Mathematical Society Colloquium! We hope that you will enjoy the conference hosted by the University of Otago. We are very grateful for sponsorship from:

- The New Zealand Mathematical Society
- ANZIAM (NZ Branch)

Extra special thanks are due to **John Shanks** for running the conference webpage and registration system, and **Leanne Kirk** from the Department of Mathematics for all the administrative help.

Chair: Florian Beyer

Local organisation committee:
Boris Baeumer
David Bryant
Jörg Hennig
Jörg Frauendiener
Fabien Montiel
Melissa Tacy

## 2 Conference venue and lecture rooms

Most of the conference is taking place in the St David Theatre Complex. The plenary talks are in the St David Lecture Theatre (STDAV) and the parallel sessions are in the St David Seminar rooms 1 – 6 (SDAV1 – SDAV6). The registration, the tea breaks and the posters are in the foyer of the St David Theatre Complex. A map of the University of Otago campus can be found here and here. The St David Lecture Theatre Complex is at the corner of Cumberland St and St David St. The Department of Mathematics & Statistics is on the second floor of the Science III Building adjacent to the St David Lecture Theatre Complex.

**WIFI access.** The networks **eduroam** and **UO_Guest** are available. In order to use the latter:

1. Select **UO_Guest** from the list of available networks on your device.
2. If a login window does not appear, open a browser and try to load any webpage.
3. Log in if you already have an account, otherwise follow the prompts to create one. When you create an account the login details are displayed on the screen. Write these down or take a snapshot for your reference. Your login details are also sent to the email address you supply.

A **UO_Guest** account provides access to the internet for up to two weeks, 500MB per day and two devices at a time.

## 3 Talks, posters and prizes

For plenary talks we have allocated 50 minutes, followed by 5 minutes for questions. For the contributed talks we have allocated 25 minutes and speakers are advised that their talks should run for 20 minutes. The plenary talks is in the St David Lecture Theatre (STDAV) and the parallel sessions take place in the St David Seminar rooms 1 - 6 (SDAV1 – SDAV6). All lecture rooms are equipped with whiteboards, computers running the Adobe Acrobat pdf reader software (connected to the internet) and projectors with VGA connectors to connect personal laptops. Laser pointers are also available.

On the Wednesday of the conference we have the Education Day. All talks on this day are devoted to the topic of maths education.

The **Aitken prize** is awarded by the New Zealand Mathematical Society for the best contributed talk by a student at the colloquium. Entrants must be enrolled (or have been enrolled) for a degree in mathematics at a university or other tertiary institution in New Zealand in 2018. The Aitken Prize honours the New Zealand-born mathematician A. C. Aitken.

Poster boards for contributed posters are available from Tuesday morning. Please bring your posters early to the Conference Opening (8.45am, STDAV). The posters should be up for the Poster Session on Tuesday at 11am in the St David Theatre Complex. The posters can stay up for the whole duration of the conference.

The **ANZIAM poster prize** is awarded to the best poster by an early career researcher (i.e., the first author of the paper must be either a student, or within 5 years of the completion of their highest degree).

These prizes as well as the NZMS Research Award, the NZMS Early Career Research Award, and Kalman Prize are awarded at the Colloquium Dinner.

## 4 Lunch and dinner options

Each conference day has a lunch break of 1h30mins. This is enough time to explore the various lunch options on or close to campus. The options are:

- St David Cafe in the St David Theatre Complex.
- University Staff Club on campus.
- Various restaurants/cafes on George Street, in particular between Albany St and Hanover St. Walking distance $\sim 15$ mins.

## 5 Events during the conference

- Registration: The registration takes place on Monday 5-7pm in the foyer of the St David Theatre Complex. Food and drinks are provided.
- Conference Opening: Professor Richard Barker, Pro-Vice-Chancellor, Sciences, is opening the conference on Tuesday at 8.45am in STDAV just before the first plenary talk by Lesley Ward.
- HoD Meeting: Tuesday lunch break in room MA229, second floor in the Science III building.
- NZMS AGM: Tuesday, 5-6pm in room SDAV1.
- ANZIAM AGM: Wednesday, 12.45-1.30pm in room SDAV1.
- NZWiM Panel Discussion: See below.
- Conference dinner: The conference dinner is on Wednesday at 7pm at Glenfalloch Restaurant & Garden. The bus picks us up at 6.15pm. The pick-up location is announced on the day.
- Excursions: Information can be found here. If you intent to go on the Pineapple Track walk, please put your name down at registration. Details about pick-up times and pick-up locations are announced on the day. Contact person: Jörg Hennig.

## 6 NZWiM Panel Discussion

The NZWiM Panel Discussion takes place on Tuesday, 6-7 in the common room, third floor of the Science III building.

Title: *Home truths for women in academic careers - also useful men!*

The panelists are:

- Associate Professor Lesley Ward (School of Information Technology and Mathematical Sciences, University of South Australia)
- Associate Professor Alex James (School of Mathematics and Statistics, University of Canterbury)

- Professor Bernd Krauskopf (Department of Mathematics, University of Auckland)
- Professor Christina Hulbe (School of Surveying, Otago University)

This session provides a forum for a frank discussion about the issues facing women in academic careers. The panelists bring a range of perspectives to this issue, including first hand experience of some of the challenges, being Head of a Department trying to address under-representation of women, chairing a university-level Equity Committee, and being involved in activities that support gender diversity and inclusivity in professional societies. All are welcome!

# 7 Timetable

## 7.1 Main programme

| | Monday | Tuesday | Wednesday | Thursday |
|---|---|---|---|---|
| 8–9 | | **Conference Opening** | | |
| 9–10 | | **Plenary Talk**<br>(STDAV)<br>Lesley Ward | **Plenary Talk**<br>(STDAV)<br>Igor' Kontorovich | **Plenary Talk**<br>(STDAV)<br>Alona Ben-Tal |
| 10–11 | | **Parallel sessions**<br>(St David Theatre Complex) | **Education Day**<br>(St David Theatre Complex) | **Parallel sessions**<br>(St David Theatre Complex) |
| 11–12 | | **Poster session (AM Tea)**<br>(St David Theatre Complex)<br>**Parallel sessions**<br>(St David Theatre Complex) | **AM Tea Break**<br>(St David Theatre Complex)<br>**Education Day**<br>(St David Theatre Complex) | **AM Tea Break**<br>(St David Theatre Complex)<br>**Parallel sessions**<br>(St David Theatre Complex) |
| 12–13 | | | **Lunch break**<br>ANZIAM AGM (SDAV1)<br>12.45-1.30 | |
| 13–14 | | **Lunch break** | **Education Day**<br>(St David Theatre Complex) | **Lunch break** |
| 14–15 | | **Plenary Talk**<br>(STDAV)<br>Geertrui Van de Voorde | | **Plenary Talk**<br>(STDAV)<br>Edgar Knobloch |
| 15–16 | | **PM Tea Break**<br>(St David Theatre Complex) | **PM Tea Break**<br>(St David Theatre Complex) | **PM Tea Break**<br>(St David Theatre Complex) |
| 16–17 | | **Parallel sessions**<br>(St David Theatre Complex) | **Education Day**<br>(St David Theatre Complex)<br>**Public Talk**<br>(STDAV)<br>Gary Sharpe | **Parallel sessions**<br>(St David Theatre Complex) |
| 17–18 | **Registration**<br>(St David Theatre Complex) | **NZMS AGM**<br>(SDAV1) | | |
| 18–19 | | **NZWiM Panel Discussion**<br>(Common room (Science III)) | | |
| 19–20 | | | **Conference Dinner**<br>(Glenfalloch Restaurant) | |
| 20–21 | | | | |
| 21–22 | | | | |

(click on event to get more info)

## 7.2 Parallel sessions programme

| | Time | Session 1 (SDAV1) | | Session 2 (SDAV2) | | Session 3 (SDAV3) | | Session 4 (SDAV4) | |
|---|---|---|---|---|---|---|---|---|---|
| | **Chair** | Markus Antoni | | Anna Barry | | Joerg Hennig | | | |
| Tue | 10.00-10.30 | Fareeda Begum * | An | Mike Steel | MB | Sebastian Schuster * | MP | | |
| Tue | 10.30-11.00 | Melissa Tacy | An | Christian Offen * | NA | Muhammad Zubair Ali Moughal * | MP | | |
| | **Chair** | Dominic Searles | | Mike Steel | | Florian Beyer | | Catherine Hassell Sweatman | |
| Tue | 11.30-12.00 | Jose Voloch | AN | Patrick Ion | OT | Liam McMahon * | MP | Giorgia Vattiato * | MB |
| Tue | 12.00-12.30 | Martin Bachraty * | AN | Alex James | OT | Joerg Hennig | MP | Hammed Olawale Fatoyinbo * | MB |
| Tue | 12.30-1.00 | Anton Baykalov | AN | Dion O'Neale | OT | Woei Chet Lim | MP | Stephen Merry * | MB |
| | **Chair** | Melissa Tacy | | Wenjun Zhang | | Cris Hasan | | | |
| Tue | 4.00-4.30 | Markus Antoni | An | Pascal Eun Sig Cheon * | IM | Andrus Giraldo | DS | | |
| Tue | 4.30-5.00 | Astrid an Huef | An | Nur Atiqah Dinon * | AD | Jana Elise Andersen-Aitken * | DS | | |
| | **Chair** | Melissa Tacy | | Alna van der Merwe | | Woei Chet Lim | | | |
| Thu | 10.00-10.30 | Dominic Searles | CO | Anna Barry | AD | Richard Laugesen | MP | | |
| Thu | 10.30-11.00 | Peter Donelan | OP | Wenjun Zhang | AD | John Butcher | NA | | |
| | **Chair** | Christopher Tuffley | | Marie Graff | | Andrus Giraldo | | David Bryant | |
| Thu | 11.30-12.00 | Michael Lockyer | GT | Winston Sweatman | AD | Bernd Krauskopf | DS | Michael Plank | MB |
| Thu | 12.00-12.30 | Sina Greenwood | GT | Bruce van Brunt | AD | Carlo Laing | DS | Catherine Hassell Sweatman | MB |
| Thu | 12.30-1.00 | Joshua Howie | GT | Alna van der Merwe | AD | Rua Murray | DS | | |
| | **Chair** | Joshua Howie | | Joerg Hennig | | Andrus Giraldo | | | |
| Thu | 4.00-4.30 | Christopher Tuffley | GT | Fabien Montiel | AD | Cris Hasan | DS | | |
| Thu | 4.30-5.00 | David Bryant | GT | Marie Graff | OP | | | | |

(click on names to get to the abstracts)

AD=Applied differential equations and modelling AN=Algebra and number theory An=Analysis CO=Combinatorics
IM=Industrial mathematics DS=Dynamical systems GT=Geometry and topology MB=Mathematical biology
MP=Mathematical physics NA=Numerical analysis OP=Optimisation OT=Other
*=Aitken prize contenstant

### 7.3 Education day programme

Please contact Cami Sawyer or David Bryant if you have further questions.

**Morning**

| Time | | Session 1 (SDAV1) | Session 2 (SDAV2) | Session 3 (SDAV3) |
|---|---|---|---|---|
| Wed | 10.00-10.30 | Jared Hockly | Gerrard Liddell | Gary Sharpe |
| Wed | 10.30-11.00 | Tanya Evans | Phillip Wilson | Peter Bier |
| | | | | |
| Wed | 11.30-12.00 | Cami Sawyer | Rachel Passmore | Chris Linsell |
| Wed | 12.00-12.30 | Julia Novak | Rachel Passmore | Jo Knox |

(click on name to get to the abstract)

**Afternoon**

| Time | | Session (SDAV1) |
|---|---|---|
| Wed | 1.30-3.30 | Engaging with Educational Issues in Mathematics Unconference |
| Wed | 4.00-4.30 | Unconference continued |

# 8 Abstracts

## 8.1 Plenary talks

### Alona Ben-Tal

(Massey University, a.ben-tal@massey.ac.nz)

*What have you done for me lately? Mathematical exploration of the cardio-respiratory system.*

The cardio-respiratory system provides a rich avenue for mathematical exploration. It consists of sophisticated neural control mechanisms with complex dynamics that affect both the breathing pattern and the heart rate. Like many other biological systems, the cardio-respiratory system has multiple processes that achieve the same function. These can occur over multiple scales (in both time and space). This makes the cardio-respiratory system challenging to model and calls for a range of mathematical approaches to analyse it. I will present three examples to illustrate different strategies that I have used to study this system. The first example will show how a new numerical method, based on an equation-free approach, can help us simplify the dynamics of a bursting neural network. The second example will describe a novel piecewise linear model that exhibits symmetry breaking. The model can explain how air flows in one direction through the lungs of birds. The third example will illustrate how optimal control theory can be used to study the interactions between the heart and the lungs of mammals. I will also discuss the significance of these examples from the viewpoints of physiologists, engineers and the general public.

**Time of the talk**: Thursday, 9am
**Room**: STDAV
**Chair**: Vivien Kirk

### Edgar Knobloch

(University of California at Berkeley, knobloch@berkeley.edu)

*Spatially localized structures in driven dissipative systems: theory and applications*

Spatially localized structures arise frequently in driven dissipative systems. In this talk I will describe a number of examples from different physical systems, followed by a discussion of the basic ideas behind the phenomenon of nonlinear self-localization that is responsible for their existence. I will illustrate these ideas using a simple phenomenological model and explain why the qualitative predictions of this model help us understand the properties of much more complicated systems exhibiting spatial localization, and specifically those arising in fluid mechanics.

**Time of the talk**: Thursday, 2.30pm
**Room**: STDAV
**Chair**: Fabien Montiel

## Igor' Kontorovich

(The University of Auckland, i.kontorovich@auckland.ac.nz)

*What mathematics is and why mathematicians who teach should be the first to care*

What is mathematics? This question has disturbed infinitely many mathematicians throughout human history, all aiming at revealing the "true" nature of our beloved subject. My point of departure in this talk is that our (often tacit) answers to this philosophical question translate into clearly visible teaching actions. Accordingly, there is a merit not just in discussing the question explicitly but also in considering possible answers in terms of their usefulness for students' learning and our teaching purposes. In the talk, I will propose conceptualizing mathematics as a historically established human discourse and discuss educational affordances of this conceptualization through examples of students' work and classroom activities.

**Time of the talk**: Wednesday, 9am
**Room**: STDAV
**Chair**: David Bryant

## Gary Sharpe

(WITT (New Plymouth), g.sharpe@witt.ac.nz)

*177,000 people can't all be wrong, or can they?*

An extensive amount of learner data exists in the Literacy and Numeracy Assessment Tool, if we dissect the numeracy data what does it tell us about:

- What sort of question are learners most likely to get wrong
- Does age, gender or ethnicity matter
- Where teaching needs to focus

We did that, let's talk about what the research shows.

**Time of the talk**: Wednesday, 4.30pm
**Room**: STDAV
**Chair**: Cami Sawyer

**Geertrui Van de Voorde**

(University of Canterbury, geertrui.vandevoorde@canterbury.ac.nz)

*Blocking sets in finite geometry*

Blocking set arose in game theory, where they were collections of players that could block every decision. From then on, blocking sets have been studied in a wide variety of settings. Whenever we have an *incidence structure* with points and lines we can speak of a *blocking set*: it is a set of points $\mathcal{B}$ such that every line contains at least one point of $\mathcal{B}$. The first structure for which blocking sets were studied is that of *finite (projective) planes.* [4]

In this talk, I will give an overview of the history of the subject. We will see that the nature of small blocking sets in finite planes is deeply connected with the algebraic structure of the plane itself. For finite Desarguesian projective planes, blocking sets can be linked with value sets of certain polynomials as done e.g. by Blokhuis [1]. I will discuss some related conjectures, in particular the linearity conjecture (see e.g. [5]).

The behaviour of small blocking sets in finite non-Desarguesian planes is intrinsically different and much less is known. I will present some recent results on the lower bound on the size of a (non-trivial) blocking set in non-Desarguesian projective and affine planes [2, 3].

## References

[1] A. Blokhuis. On the size of a blocking set in PG$(2, p)$. *Combinatorica* 14, 111–114, 1994.

[2] J. De Beule, T. Héger, T. Szőnyi, and G. Van de Voorde. Blocking and double blocking sets in finite planes. *Electronic J. Combin*, 23(2), P2.5, 2016.

[3] M. De Boeck and G. Van de Voorde. A new lower bound on the size of an affine blocking set. To appear in *Electronic J. Combin.*

[4] M. Richardson. On finite projective games. *Proc. Amer. Math. Soc.,* 7, 458-465, 1956.

[5] P. Sziklai. On small blocking sets and their linearity. *J. Combin. Theory Ser. A*, 115(7), 1167–1182, 2008.

**Time of the talk**: Tuesday, 2.30pm
**Room**: STDAV
**Chair**: Felipe Voloch

**Lesley Ward**
(University of South Australia, lesley.ward@unisa.edu.au)

*Harmonic analysis on spaces of homogeneous type*

What do the integers, the Sierpiński gasket, compact Riemann surfaces and the Heisenberg group have in common? Each of them is a *space of homogeneous type* $(X, d, \mu)$: a set $X$ equipped with a way of measuring the distance between any two points (a *quasi-metric* $d$) and a suitable way of measuring the volume of reasonable subsets of the set (a *doubling measure* $\mu$). A familiar example is Euclidean space $\mathbb{R}^n$ equipped with the Euclidean metric and Lebesgue measure. Spaces of homogeneous type arise in a variety of contexts, including Riemannian geometry, functions of several complex variables, and Lie theory.

The Calderón–Zygmund theory in harmonic analysis deals with singular integral operators and the functions on which they act. Early impetus for the theory came from problems in partial differential equations and Fourier theory. Much effort has been devoted to generalising the Calderón–Zygmund theory in several directions. Here we focus on the generalisation from functions defined on Euclidean spaces $\mathbb{R}^n$ to functions defined on spaces $X$ of homogeneous type. In particular, for general $X$ the Fourier transform and the group structure of $\mathbb{R}^n$ are missing.

The goal is to build a Calderón–Zygmund theory on spaces of homogeneous type that is as complete as it can be, recovering the classical results where possible and finding appropriate replacements or analogues where needed. I will survey some recent progress towards this goal.

**Time of the talk**: Tuesday, 9am
**Room**: STDAV
**Chair**: Melissa Tacy

### 8.2 Talks

**Jana Elise Andersen-Aitken**
(University of Auckland, elle.musoke@auckland.ac.nz)

*A surface of heteroclinic connections in a four-dimensional slow-fast system*

Slow-fast dynamical systems are systems in which some variables change faster than others. From chemical reactions to electric circuits, examples of slow-fast systems are found in a wide variety of disciplines. By reason of their ubiquity, phenomena that arise from the multiple-time-scale nature of slow-fast dynamical systems are of great interest. In particular, we study mixed-mode oscillations (MMOs), periodic orbits that have segments of low amplitude oscillations and segments of high amplitude oscillations. Mechanisms for generating MMOs in four-dimensional slow-fast systems are largely unexplored. We consider the prototypical four-dimensional Olsen model that has an MMO. Near the MMO, Fenichel theory guarantees the existence of two one-dimensional, so-called saddle slow manifolds. One saddle slow manifold has a three-dimensional stable and a two-dimensional unstable manifold and the other has a two-dimensional stable and a three-dimensional unstable manifold. Numerical continuation methods are used in conjunction with appropriately defined boundary-value problems and Lin's method to compute a two-dimensional surface of heteroclinic connections formed by the intersection of the two three-dimensional manifolds. This surface of heteroclinic connections is an important ingredient for the formation of the MMO.

**Astrid an Huef**
(Victoria University of Wellington, astrid.anhuef@vuw.ac.nz)

*Operator algebras of quasi-lattice ordered groups*

Let $G$ be a discrete group with a generating submonoid $P$ such that $P \cap P^{-1} = \{e\}$. There is a partial order on $G$ defined by $x \leq y$ if and only if $x^{-1}y \in P$. The pair $(G, P)$ is quasi-lattice ordered if all $x, y \in G$ with a common upper bound in $P$ have a least upper bound in $P$.

Quasi-lattice ordered groups and their Toeplitz algebras were introduced by Nica in 1992. I will discuss some examples of interest, including the Baumslag-Solitar group

$$G = \langle a, b : ab^c = b^d a \rangle$$

with submonoid $P$ generated by $a$ and $b$. Then I will discuss sufficient conditions on the quasi-lattice ordered group for operator-algebraic properties of interest.

**Markus Antoni**

(Karlsruhe Institute of Technology (KIT), markus.antoni@kit.edu)

*Maximal regularity for stochastic evolution equations with memory*

In this talk we give a brief introduction to stochastic evolution equations with memory driven by a Gaussian white noise and the maximal regularity of their solutions. Whereas this is well-known for the special case of a Riesz kernel, not much is known for arbitrary kernels. Using merely an $L^1$ estimate of the corresponding scalar ordinary differential equation, we use functional calculus results from spectral theory to enhance all estimates to the operator-valued setting. In the end, this approach leads to unique mild solutions with optimal regularity properties, in particular Hölder continuous trajectories. This is joint work with Boris Baeumer and Petru Cioica-Licht.

**Martin Bachraty**

(University of Auckland, mbac631@aucklanduni.ac.nz)

*Approaching the Moore bound for diameter 3 with Cayley graphs*

The order of a graph of degree $d$ and diameter $k$ cannot exceed the Moore bound, which has the form $M(d,k) = d^k - O(d^{k-1})$ for $d \to \infty$ and fixed $k$. Let $\mathrm{Cay}(d,k)$ be the largest order of a Cayley graph of degree $d$ and diameter $k$. It was shown in 2012 by Šiagiová and Širáň that $\limsup_{d\to\infty} \mathrm{Cay}(d,k)/M(d,k) = 1$ for $k = 2$. We use results on finite geometries, in particular generalized quadrangles, to show that this is true also for $k = 3$.

**Anna Barry**

(University of Auckland, anna.barry@auckland.ac.nz)

*Three-vortex collapse in the surface quasigeostrophic equations*

The classical point vortex model derives from the two-dimensional Euler equations and results in a Hamiltonian system of ODE. An analogous derivation applied to the surface quasi-geostrophic (SQG) equations results in another point vortex model with a slightly different interaction law. In this talk, we focus on three-vortex collapse in the SQG model. In particular, we find that the collapse point can be approached in either a self-similar or non-self-similar fashion. Self-similarity occurs only when the Hamiltonian is zero, while non-self-similarity appears for non-zero values of the same. For both cases, collapse is allowed for any choice of circulations within a permitted interval. These results differ strikingly from the classical point vortex model, where collapse is self-similar for any value of the Hamiltonian, but the vortex circulations must satisfy a strict relationship.

**Anton Baykalov**
(The University of Auckland, abay417@aucklanduni.ac.nz)

*Intersection of conjugate solvable subgroups in GL(n, q)*

Assume that a finite group $G$ acts on a set $\Omega$. A point $\alpha \in \Omega$ is $G$**-regular** if the stabilizer of $\alpha$ is trivial. Define the action of the group $G$ on $\Omega^k$ by

$$g : (\alpha_1, \dots, \alpha_k) \mapsto (\alpha_1 g, \dots, \alpha_k g).$$

Let $G$ act faithfully and transitively on $\Omega$, for a positive integer $m$ denote the number of $G$-regular orbits on $\Omega^m$ by $Reg(G, m)$. If $H$ is a subgroup of $G$ and $G$ acts by the right multiplication on the set $\Omega$ of right cosets of $H$ then $G/H_G$ acts faithfully and transitively on $\Omega$. (Here $H_G = \cap_{g \in G} H^g$.) In this case, we denote $Reg(G/H_G, m)$ by $Reg_H(G, m)$.

In this work we consider Problem 17.41 b) from the "Kourovka notebook":

*Let $H$ be a solvable subgroup of a finite group $G$ that has no nontrivial solvable normal subgroups. Do there always exist five conjugates of $H$ whose intersection is trivial?*

The problem is reduced to the case when $G$ is almost simple by E. P. Vdovin (2012). In particular it sufficient to show that for every solvable subgroup $H < G$

$$Reg_H(G, 5) \geq 5$$

for all almost simple groups $G$.

The main statement of the present work is the following.

**Theorem.** *Let $(n, q)$ be such that $G = GL(n, q)$ is not solvable. If $S$ is a maximal solvable subgroup of $G$ then $Reg_S(G, 5) \geq 5$.*

**Fareeda Begum**
(Canterbury University, fareeda.begum@pg.canterbury.ac.nz)

*Quadratic means of univalent holomorphic mappings on an annulus*

We discuss counterparts of the Schwarz lemma for univalent holomorphic mappings on an annulus. We have some examples to show that a pointwise version of the Schwarz lemma for univalent holomorphic on an annulus would not be possible. We also determine an integral means version of the Schwarz lemma for univalent holomorphic mappings of an annulus. It is found that normalization and univalence both play a significant role.

**Peter Bier**
(University of Auckland, p.bier@auckland.ac.nz)

*Academics anonymous bring and share session*

Come along to a supportive environment, prepared to share for 1-2 minutes about an idea or experience relating to your teaching of tertiary mathematics. This could be something that worked well for you or something that bombed. Participants would love to beg/borrow/steal your good ideas and will also learn a lot from things that didn't pan out how you hoped. Remember we are all in this together, so sharing failures can be just as valuable as sharing success stories and sometimes has more impact!

**David Bryant**
(University of Otago, david.bryant@otago.ac.nz)

*Evolution and inner products for convex sets*

The environmental niche of a species is often modeled as a convex set in some abstract space. We have been using techniques based on species distribution data and physiological models to infer niches for different species. Our interest has been in studying how those niches evolved. The associated computational challenges has necessitated the development of new mathematical tools for inferring, and approximating, convex sets. In particular, we have defined a form of inner product for (closed, bounded) convex sets. I'll talk about properties and characterisations of this "set inner product" and show how ancestral niches can be inferred with a little bit of functional analysis.

**John Butcher**
(University of Auckland, butcher@math.auckland.ac.nz)

*Space saving Runge–Kutta methods*

The fourth order Runge–Kutta method

$$\begin{array}{c|cccc}
0 & & & & \\
\frac{1}{2} & \frac{1}{2} & & & \\
\frac{1}{2} & 0 & \frac{1}{2} & & \\
1 & 0 & 0 & 1 & \\
\hline
 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6}
\end{array}$$

is widely used in the numerical solution of ordinary differential equation systems. It was pointed out by S. Gill: Proc. Cambridge Philos. Soc. **47** (1951), pp 96–108, that for an $N$ dimensional system, a memory size of $4N$ is required to implement the method. Gill derived a method in which this is reduced to $3N$. This talk will outline Gill's analysis and the derivation of his method and survey some of the forward-thinking contributions of this pioneering computational scientist. Finally, a new method will be presented which achieves Gill's aims in a different way.

**Pascal Eun Sig Cheon**
(University of Auckland, eche291@aucklanduni.ac.nz)

*Domain truncation in pipeline monitoring problems*

On-line pipeline process monitoring is a common problem faced in industrial applications. The aim is to reconstruct the evolution of liquid concentration inside a segment of a pipe. A typical measurement modality for this is the electrical impedance tomography (EIT). This is a non-invasive boundary measurement modality which maps from the unknown concentration distribution space to the voltage measurement space. When modelling the measurement process, the computational domain is chosen so that all current flows in the object is covered. Consequently, the domain may include a nuisance subdomain which is not of our reconstruction interest. The nuisance domain can be artificially truncated out by introducing a non-local map called the Dirichlet-to-Neumann (DtN) map. The DtN map is used to formulate the boundary condition along the truncation boundary. In this talk, we propose a way of incorporating the domain truncation in the time-variant problems. The viability of this approach in on-line pipeline process monitoring problem is demonstrated with numerical simulations.

**Nur Atiqah Dinon**
(The University of Auckland, nur.dinon@auckland.ac.nz)

*Visitors from beyond Neptune*

The Kuiper Belt (KB) is a region that lies outside the orbit of Neptune. This region is made of possibly millions of small icy bodies known as Kuiper Belt Objects (KBOs) that revolve around the Sun. The first KBO was discovered in 1992 and over 1000 are now known. Some KBOs are comparable to Pluto in size, the rest are smaller than Pluto. Given that the KB is located so far away and extremely difficult to observe, the dynamics of the KB is still poorly understood. Several studies have been done to explain the formation of the KB and its structure. However, there has been little work on the long-term evolution of the KB. Of particular interest in the long-term evolution is what percentage of the KBOs is transported into the inner Solar System which is the region inside Jupiter's orbit. KBOs in the inner Solar System have the potential to hit Earth. In this talk I will present the numerical simulations on real data of KBOs obtained from Minor Planet Center (MPC) over 100 million years, showing how many of the test particles were cast out of the Solar System, the number of possible collision and number of particles remained at the end of simulations.

**Peter Donelan**
(Victoria University of Wellington, peter.donelan@vuw.ac.nz)

*Hyperbolic generalised inverses, optimality and hybrid control*

Moore–Penrose generalised inverses can be defined when the inner product on the domain and/or range of a matrix transformation is indefinite. However, unlike in the Euclidean case, these inverses do not necessarily exist. The Lie algebra, $se(3)$, of the Euclidean group, which plays a fundamental role in robot kinematics, has a one-parameter family of invariant hyperbolic forms. These give rise to corresponding generalised inverses for the Jacobian of robot arm kinematics, relating joint velocities to end-effector velocities in $se(3)$. We explore existence conditions for these inverses and show how they relate to optimal solutions for the inverse kinematics, as well as projection operators used in hybrid control systems.

**Tanya Evans**

(University of Auckland, t.evans@auckland.ac.nz)

*When on-line pre-lecture quizzes are integrated into a course: Exploring the impact of a small-scale change in an instructional model in one large-scale mathematics course.*

Presenters: Tanya Evans (Lecturer in Mathematics Education, Department of Mathematics, University of Auckland) and Julia Novak (Associate Dean of Teaching and Learning of Faculty of Science, University of Auckland)

Driven by the advances of emerging technologies, the higher education sector is forced to move with the times and look for ways to reduce operating costs and become sustainable in a technological era. There is currently a great deal of buzz within higher education about the introduction of blended-learning, lecture recordings and flipped lectures. New technologies are guiding the expectations of the students and questioning the need for, and the value of, face-to-face lectures.

In this presentation, we will talk about a small-scale change in an instructional model for delivery of a university mathematics course at stage II which covers Calculus, Linear Algebra and Ordinary Differential Equations. As part of the project, a bank of multi-choice questions was developed and delivered via on-line quizzes preceding every lecture, each containing two short multi-choice questions. The implementation of this new resource not only provided instant feedback to students but also enabled teachers to access data prior to every lecture informing them about overall student learning of the material from the previous lecture and afforded opportunity to adjust their lecturing - content-wise and time spent on revision. The simplicity of our model is its characterizing feature: the core driver is the frequent pre-lecture assessment that contributes to the final grade for the course. As evidenced by our data, majority of students embraced this blended-learning mode of learning and that, in turn, led to increased attendance of live lectures and overall improvement in students' engagement with the learning practices.

**Hammed Olawale Fatoyinbo**

(Massey University (Manawatu Campus), h.fatoyinbo@massey.ac.nz)

*Emergence of spatiotemporal patterns in pacemaker coupled excitable cells*

Excitable cells such as smooth muscle cells and neurons can generate action potentials when external inputs are imposed on them. Spatial propagation of action potentials between cells plays a fundamental role in human physiology. Theoretically, these activities can be modelled by systems of reaction-diffusion equations. We formulated a one-dimensional reaction-diffusion model to explore pattern formation in excitable media. We first consider the time-dependent system with no diffusion to analyse the stability of the steady states using phase plane and bifurcation analysis. Our analysis shows that the system can transition between a stable regime to an oscillatory regime, and vice versa via different kinds of bifurcation. In my talk, I will discuss the emergence of spatiotemporal patterns in coupled excitable cells and role that these bifurcations play.

**Andrus Giraldo**
(The University of Auckland, agir284@aucklanduni.ac.nz)

*Dynamic complexity in two coupled photonic crystal nanocavities*

Nonlinear optical resonators are the subject of much interest due to the plethora of effects that they may generate, including optical frequency combs and cavity solitons. In particular, introducing a small gap into a Photonic Crystal creates an optical resonator with extremely small mode volumes. Such nano-devices offer the possibility of doing nonlinear optics with very small photon numbers, threshold-less lasing and many other intriguing experiments in both classical and quantum optics.

We consider an optical device consisting of two coupled nanocavities in a Photonic Crystal that are optically driven. This experimental system has been shown to exhibit spontaneous symmetry breaking and bistable behaviour. However, its more complex dynamics have not yet been characterised. Interestingly, the overall behaviour of this type of device is captured by a four-dimensional vector field model. We conduct a bifurcation analysis to determine the dynamics that arises when the intensity and frequency of the optical input are varied, while the other parameters of the Photonic Crystal are fixed to experimental values. In particular, we find Shilnikov bifurcations of the wild type, and transitions between chaotic attractors with different symmetry properties. Hence, this four dimensional system allows us to study and visualize the consequences of higher-dimensional chaos in a concrete applied model arising from an application. We present and characterise how these global bifurcations are organised around bifurcations of higher codimension. In particular, we also discuss the transition, from simple to chaotic dynamics, between different types of chaotic attractors.

**Marie Graff**
(University of Auckland, marie.graff@auckland.ac.nz)

*Adaptive eigenspace method for inverse scattering problems in the frequency domain*

We consider inverse medium problems for the Helmholtz equation. The time-harmonic inverse problem is formulated as a PDE-constrained optimization problem and solved by an inexact truncated Newton-type iteration. Instead of a grid-based discrete representation, the unknown velocity is projected to a particular finite-dimensional basis of eigenfunctions, which is iteratively adapted during the optimization, introducing regularization into the inversion and omitting standard Tikhonov-type regularization terms. Both analytic and numerical evidence underpins the accuracy of the Adaptive Eigenspace representation.

**Sina Greenwood**
(University of Auckland, sina@math.auckland.ac.nz)

*A characterisation of the dimension of inverse limits of set-valued functions on intervals*

Inverse limits have been studied intensively, and have been broadly applied. For example, they have played a vital role in the development of continuum theory since extremely complicated spaces can be obtained as inverse limits of simple bonding functions on very simple spaces.

Generalized inverse limits (GILs), or inverse limits with set-valued functions, were recently introduced by Ingram and Mahavier and provide a valuable extension to applications of inverse limits. In continuum theory, for example, Kennedy and Nall have developed a simple method for constructing families of $\lambda$-dendroids from GILs on closed intervals with simple bonding maps.

GILs behave very differently to inverse limits and their study is rapidly gaining momentum. The dimension of an inverse limit, is one example of difference. The dimension of an inverse limit is the supremum of the dimensions of its factor spaces, whereas GILS can have any dimension, even GILs on intervals.

For a sequence $\boldsymbol{f}$ of upper semicontinuous set-valued functions $f_i : \mathbb{I}_i \to 2^{\mathbb{I}_{i-1}}$ (where $\mathbb{I}_i = [0,1]$ for each $i \in \mathbb{N}$), we introduce the notion of a weighted sequence. We show that $\dim\left(\varprojlim \boldsymbol{f}\right) = n$ if and only if the maximal length of a weighted sequence admitted by $\boldsymbol{f}$ is $2n-2$. Furthermore, $\dim\left(\varprojlim \boldsymbol{f}\right) \geq n$ if and only if there is an increasing sequence $\langle i_1, \ldots, i_n \rangle$ such that the projection of $\varprojlim \boldsymbol{f}$ to $\prod_{1 \leq j \leq n} \mathbb{I}_{i_j}$ contains an $n$-cell.

**Cris Hasan**

(University of Auckland, rhas033@aucklanduni.ac.nz)

*Existence and stability of periodic travelling waves: who will prevail in a Rock-Paper-Scissors game?*

We study a Rock-Paper-Scissors model that describes the spatiotemporal evolution of three competing populations, or strategies, in evolutionary game theory and biology. The dynamics of the model is determined by a set of partial differential equations (PDEs) that features travelling waves (TWs) in one spatial dimension and spiral waves in two spatial dimensions. We focus on periodic (in space) TWs, a phenomenon that is also inherently found in other reaction-diffusion models. The existence of periodic TWs can be established via the transformation of the PDE model into a system of ordinary differential equations (ODEs) under the assumption that the wave speed is constant. We explore the bifurcation diagram of the ODE system and investigate the existence of TWs as different parameters are varied. Determining the stability of periodic TWs is more challenging and requires a study of the essential spectrum of the linear operator of the periodic TWs. Based on the continuation scheme developed in [Rademacher et al., Physica D, 2007], we compute this spectrum and the curve of instability. We also build on this scheme and develop a method for computing what we call belts of instability, which are indicators of the temporal expansion rates of unstable TWs. We finally show how these results compare with direct simulations of the PDE model.

**Catherine Hassell Sweatman**

(Auckland University of Technology, catherine.sweatman@aut.ac.nz)

*Mathematical model of diabetes and lipid metabolism : investigating the timescales corresponding to both low carbohydrate diets and low fat diets for weight loss*

Currently there is great debate on the relative merits and dangers of fats and sugars in our diet. Both low carbohydrate and low fat diets are recommended for those who wish to lose weight and avoid Type II diabetes. In order to investigate the relative merits of different diets, a mathematical model of glucose, insulin, glucagon, $\beta$-cell, leptin and fat dynamics and hepatic, peripheral and adipose insulin sensitivity is presented. The model includes plasma non-esterified fatty acids, muscle lipids, hepatic lipids and very low density lipoprotein triglycerides, allowing comparisons with recent health research. Decay constants for fat mass are predicted to vary widely, depending on the diet. Implications for experimental work are discussed.

**Joerg Hennig**
(University of Otago, jhennig@maths.otago.ac.nz)

*Gowdy-symmetric cosmological models in Einstein-Maxwell theory*

Gowdy-symmetric spacetimes are useful test cases for studying key issues in mathematical relativity and cosmology. We introduce a new class of Gowdy-symmetric models as solutions to the Einstein-Maxwell equations in electrovacuum. These can be considered to be nonlinear perturbations, through an electromagnetic field, of certain previously studied vacuum models. Utilising methods from soliton theory, we analyse the effects of the Maxwell field on global properties of the solutions. In particular, we show existence of regular Cauchy horizons, i.e. hypersurfaces beyond which causality and determinism break down. Moreover, we investigate special singular cases in which curvature singularities form.

**Jared Hockly**
(Western Springs College/University of Auckland, jhockly@gmail.com)

*Perspectives of a secondary teacher teaching at university*

I have had the opportunity to teach and study at the University of Auckland's Mathematics Department in 2018 through a teaching fellowship. I can share some interesting observations of the differences and similarities in mathematics education in secondary and tertiary settings. Some interesting points to cover: what we teach, assessment, lecturing vs classroom teaching, and video lectures.

**Joshua Howie**
(Monash University, josh.howie@monash.edu)

*Exceptional fillings of weakly generalised alternating knots*

Every closed 3-manifold can be obtained by Dehn filling a link in the 3-sphere. Thurston's hyperbolic Dehn surgery theorem states that all but finitely many such fillings on a hyperbolic knot complement admit a hyperbolic structure, and it remains an open problem to classify the exceptions. Recently the exceptional fillings of alternating knot complements in the 3-sphere have been completely classified. We study knots in 3-manifolds which have alternating diagrams onto higher genus surfaces, and under certain conditions on the diagram, we show that all non-trivial fillings are hyperbolic. Joint work with Jessica Purcell.

**Patrick Ion**

(GDML / U Michigan, pion@umich.edu)

*The Global Digital Mathematics Library and the International Mathematical Knowledge Trust*

Since being set up in 2014 by the International Mathematical Union the Global Digital Mathematics Library (GDML) WG, has been working toward a future World Heritage DML, as the US NAS symposium of 2012 put it. Some of the reasons why we want this and do not yet have it will be discussed. The founding of the International Mathematical Knowledge Trust (IMKT) in Waterloo ON, Canada is an important move in the right direction. The IMKT's form and initiatives will be described, with attention to why the grand project is the where it is.

**Alex James**

(University of Canterbury, alex.james@canterbury.ac.nz)

*Gender and Societies*

Are our science societies diverse and representative of their communities? Are there ways to improve diversity? We explore these ideas using a mixture of data and stochastic models.

**Jo Knox**

(University of Auckland, joknox@hotmail.co.uk)

*Is Mathematical Proof, Childproof?*

Mathematical proof is fundamental to the work of mathematicians and yet it is a process that has been marginalised in schools, particularly in primary school classrooms. In this session, I will outline reasons that may explain the absence of proof in classrooms. I will also discuss my current PhD research titled, "Is mathematical proof, childproof?" where I aim to develop tasks that support young students to engage in proving activity and develop valid reasoning.

## Bernd Krauskopf

(University of Auckland, b.krauskopf@auckland.ac.nz)

*Pulse train dynamics in an excitable laser with delayed optical feedback*

Recent experiments demonstrated that an excitable VCSEL micropillar laser with delayed optical feedback is able to sustain trains of optical pulses. These can be triggered largely independently by optical perturbations and sustained simultaneously in the external cavity. A bifurcation analysis of a rate-equation model in the form of a system of delay differential equations shows that the the only attractors are periodic solutions with different numbers of equally spaced pulses. Hence, although coexisting pulse trains can seem independent on the timescale of the experiment, they correspond to extremely long transient dynamics. The switching dynamics of the device are explained by studying the associated basins of attraction.

This is joint work with Soizic Terrien, Neil G.R. Broderick and Sylvain Barbay.

## Carlo Laing

(Massey University, c.r.laing@massey.ac.nz)

*Reduced models for networks of model neurons*

Directly simulating large networks of neurons is time-consuming and does not provide much insight. However, for large networks of "theta neurons" it is possible to derive reduced models that are valid under some weak assumptions. I will show how to derive several such models of differing complexity and accuracy, and how these models can be used to efficiently investigate the effects of a network's structure on its dynamics.

## Richard Laugesen

(University of Illinois, Laugesen@illinois.edu)

*From Neumann to Steklov and beyond, via Robin: the Weinberger way*

Nature loves symmetry, especially when optimizing energies and frequencies. Weinberger showed the first frequency of a free membrane (eigenvalue of the Neumann Laplacian) is maximal for the disk, among regions of given area. Brock did the same for membranes with mass concentrated at the boundary (the Steklov eigenvalue). We connect these two results by generalizing to eigenvalues of the Laplacian with Robin boundary conditions.

## Gerrard Liddell

(Retired Otago, gliddell@maths.otago.ac.nz)

*Manipulating Mathematics*

Trigonometry is regarded as the most daunting NCEA topic as it requires both algebraic and geometric concepts and even in the Scholarship exam it is presented as no less than 28 formulae. Shifting the focus to the essential concepts and the systematic processes relating them, explored via concrete manipulations, makes the trigonometry more accessible while reducing the burden of rote memory. If trigonometry is seen as simple, intuitive and useful,it will be retained more easily.

Similarly simple manipulations in Mathematica that reduce complex ideas to simple intuitions will be demonstrated.

## Woei Chet Lim

(University of Waikato, woeichet.lim@waikato.ac.nz)

*Monotonic functions in Bianchi models with a scalar field*

The dynamics of Bianchi cosmological models are described by a system of ODEs. Qualitative results such as attractors can be obtained. The Hamiltonian provides a natural monotonic function that may be helpful in isolating the attractors. I will illustrate this with examples, and point out the limitations of this method.

## Chris Linsell

(University of Otago, chris.linsell@otago.ac.nz)

*Innumeracy of first year undergraduate students*

We used the TEC Adult Numeracy Assessment Tool to establish the level of numeracy competency of students in a range of first year papers. Our results show that there is a significant proportion of students with numeracy levels below that expected of school Year 9 students. We will discuss the serious impact this is likely to have on students' chances of academic success.

**Michael Lockyer**

(University of Auckland, michael.lockyer@auckland.ac.nz)

*The mountain climbing problem and paths in set valued inverse limits*

Informally, the mountain climbing problem asks 'Can two people climbing a mountain along different tracks reach the summit, while staying at the same altitude as each other at every point in time?'. More formally, it can be stated: 'Given continuous, surjective functions $f, g : [0,1] \to [0,1]$, do there exist continuous functions $f', g' : [0,1] \to [0,1]$ such that $ff' = gg'$?'.

The inverse limits of set valued functions are a topological structure that have recently gained a lot of attention due to their rich structure and largely unknown properties. It turns out the mountain climbing problem is central to determining whether the inverse limits of set valued functions are path connected. In this talk I will introduce set valued inverse limits, and describe some conditions relating to path connectedness, including the relationship to the mountain climbing problem.

**Liam McMahon**

(University of Waikato, lcm21@students.waikato.ac.nz)

*A solar flare model: Temporal evolution of linear magnetic reconnection*

Magnetic reconnection is a model for energy release that predicts rates too slow to explain the rapid release of energy observed in solar flares. Hence, we turn to a linear model that predicts that small amounts of magnetic energy can be liberated at very fast rates for purely resistive reconnection. However, the introduction of viscous, Hall, electron inertia, axial or pressure effects have been shown to stall fast linear reconnection. Thus, it appears that linear reconnection is not viable as a mechanism for fast energy release. An alternative explanation has been proposed that the viscous dissipation during linear reconnection can actually provide an avenue for fast energy release which we demonstrate here to be, significantly, unimpeded by pressure effects. Additionally, we introduce axial effects in the form of a Quasi-Separatrix Layer, which was previously shown to stall reconnection, and a 3D null point, which we show to be able to support purely resistive fast reconnection using analytical arguments and numerical simulations.

### Stephen Merry

(University of Canterbury, stephen.merry@pg.canterbury.ac.nz)

*Using individual contacts to make weighted networks*

The Common Brushtail Possum is an introduced species that, over the last 150 years, has caused significant damage to New Zealand's native flora and fauna. Moreover, Brushtail Possums are one of the major wildlife reservoirs of Bovine Tuberculosis, and are thought to be one of the main vectors of the disease to cows.

Motivating the research is a set of data, collected by Dr Dan Tompkins and his team at Landcare Research, detailing the interactions between 4 groups of approximately 40 possums over a 6 month period. We construct an individual-based model, simulate the movements of and interactions between a group of individuals, and produce probabilistic results about the frequency and duration of their interactions. These results are then used to estimate relevant parameters of the possum populations in the aforementioned data. We use the parameters to construct weighted networks representing interactions between these possums, on which we will simulate the transmission of Bovine TB.

### Fabien Montiel

(University of Otago, fmontiel@maths.otago.ac.nz)

*Near trapping of water waves by a two-dimensional resonator*

The resonant behaviour of a cylindrical water wave resonator partially slitted along its vertical dimension is investigated in the context of linear time-harmonic ocean wave theory. The resonant structure serves as a model of oscillating water column device to extract energy from ocean waves. An integral equation/Galerkin method is devised to compute the field scattered by the structure in a modal basis of cylindrical eigenfunctions. Numerical results suggest the existence of a near trapped mode at low frequency, which can be identified as a nearby pole of the scattering coefficients in the complex frequency plane. We then use the poles themselves in a Blaschke product to approximate the spectral response of the resonator.

### Muhammad Zubair Ali Moughal

(University of Waikato, zubairmoughaljobs100@yahoo.com)

*Generating spiky solutions of Einstein field equations with the Geroch transformation*

In general relativity, the dynamics of spacetime is described by the Einstein field equations, which are a set of partial differential equations. The Geroch transformation is a solution-generating transformation and may generate spiky solutions. Most spiky solutions generated so far form spikes at early times. In this talk I present a solution that forms spikes at late times

**Rua Murray**
(University of Canterbury, rua.murray@canterbury.ac.nz)

*Random dynamical systems and transfer operator cocycles*

Random dynamical systems and transfer operator cocycles

Random dynamical systems are generated by recursively applied sequences of maps, where the choice of map at each timestep is determined by a stochastic process. Such systems are usually formulated as a skew-product, in which the “base” dynamics is autonomous and the “fibre-to-fibre” mappings are determined by the base. An observer watching only the fibres sees non-autonomous dynamics, or “random” orbits. Typical questions of interest relate to the long-term distribution of orbits, mass-transport, rates of mixing and so on, and there are numerous real-world applications.

This talk will introduce the important ideas for studying random dynamics from an ergodic theory viewpoint. Questions of “stochastic stability” can be formulated (and answered) in this way, and much of the theory works as one might expect when the base process is IID. In such cases, insight can even be gained via a transfer operator obtained by averaging over all fibres. When the base process is not IID (for example, an ergodic dynamical system), averaging may yield irrelevant objects, and one must study cocycles of transfer operators. Important dynamical structures on fibres become random variables. This picture will be outlined, and some positive results on accessing the distribution of orbits of certain random interval maps will be given.

**Julia Novak**
(The University of Auckland, j.novak@auckland.ac.nz)

*Renegotiating the didactic contract: Understanding and managing student expectations about your teaching and their learning. (First Year in Mathematics Discussion) – Part 2*

Abstract: A structured discussion exploring the relationship between student learning and the didactic contract. We will identify practical solutions for school teachers and university lecturers to renegotiate the didactic contract and enable deeper learning without increasing student anxiety and teacher stress.

First Year in Maths (FYiMaths) is a network of mathematicians teaching in universities in Australia and New Zealand. The goal of the group is to improve outcomes for students in undergraduate mathematics courses by sharing and developing teaching practices and building connections between secondary schools and universities.

About us: Julia Novak and Cami Sawyer have established the NZ group of FYiMaths because they are interested in furthering conversations around teaching and learning. Julia is a professional teaching fellow at the University of Auckland and the Associate Dean (Teaching and Learning) for the Faculty of Science. Cami is a senior tutor at Massey University.

**Christian Offen**
(Massey University, Manawatu, c.offen@massey.ac.nz)

*Symplectic integration of PDEs using Clebsch variables*

Many PDEs (Burgers' equation, KdV, Camassa-Holm, Euler's fluid equations,...) can be formulated as infinite-dimensional Lie-Poisson systems. The Poisson structure is connected to conservation properties and other geometric features of solutions to the PDE. It is, in a sense, the underlying mathematical reason for symmetries and conservation laws of the PDE and, therefore, of great interest for numerical integration. However, it can be difficult to numerically capture this important piece of structure. Can symplectic integrators help?

**Dion O'Neale**
(University of Auckland, d.oneale@auckland.ac.nz)

*Constructing socio-political networks from obsidian artefacts in pre-European Aotearoa/New Zealand*

The Polynesian colonists who settled New Zealand some 700 years ago, brought with them cultural conceptions of chiefdom based on genealogical affiliation (whakapapa) and territory (mana whenua). It has been suggested that the initial settlers lived in relatively autonomous villages, and that over centuries these grew to form geographically larger social units (hapū), which eventually coalesced into tribal groups known as iwi. We have used archaeological records to construct networks of obsidian movements in pre-European Aotearoa New Zealand, and to investigate factors that may have influenced how iwi groups gathered resources, be they geographic, economic, or social. We create a bipartite network of obsidian source locations and the archaeological study sites where the artefacts were ultimately found. Analysis of the spatial and temporal aspects of the source-site bipartite network is used to provide insight into the movement and interactions of the groups who were collecting, transporting, and using the obsidian. The bipartite networks allow us to test various hypotheses that might explain the unique distribution of obsidian throughout the Northland and Auckland regions of Aotearoa/New Zealand. Using tools such as similarity measures and community detection we identify those regions with similar patterns of obsidian sourcing which we use to infer social networks in pre-European Aotearoa/New Zealand. (Joint work with Caleb Gemmell and Thegn Ladefoged)

## Rachel Passmore

(University of Auckland, r.passmore@auckland.ac.nz)

*Promoting relationships between the secondary and tertiary sectors*

The University of Auckland's Department of Mathematics is involved with a number of initiatives to promote links with the wider Mathematics community. Some initiatives are established and have been running for several years, some are brand new. In this talk I will describe these initiatives which include competitions, exchange programmes and resource development. I will contrast our approach with that used by ACU (Australian Catholic University) and a UK initiative which I hope to build on here in New Zealand. Feedback on approaches used in other regions will be welcomed.

## Rachel Passmore

(University of Auckland, r.passmore@auckland.ac.nz)

*Outreach sharing*

TBA

## Michael Plank

(University of Canterbury, michael.plank@canterbury.ac.nz)

*Mitigating fisheries-induced evolution*

An integro-partial differential equation has been used to model multi-species fish communities and their response to fishing. This talk will present some results from the model about the evolutionary selection pressures generated by different fishing patterns. The conclusion is that protecting large individuals from fishing is key to mitigating the effects of fisheries-induced evolution.

## Cami Sawyer

(Massey University, c.sawyer@massey.ac.nz)

*Renegotiating the Didactic Contract: Understanding and managing student expectations about your teaching and their learning. (First Year in Mathematics Discussion) – part 1*

We will explore the established norms of the didactic contract between students and teachers, in school mathematics, and between students and lecturers, in university mathematics. Following this we will consider how the differences in these contracts may affect the transition of students from school to university.

First Year in Maths (FYiMaths) is a network of mathematicians teaching in universities in Australia and New Zealand. The goal of the group is to improve outcomes for students in undergraduate mathematics courses by sharing and developing teaching practices and building connections between secondary schools and universities.

About us: Julia Novak and Cami Sawyer have established the NZ group of FYiMaths because they are interested in furthering conversations around teaching and learning. Julia is a professional teaching fellow at the University of Auckland and the Associate Dean (Teaching and Learning) for the Faculty of Science. Cami is a senior tutor at Massey University.

## Gary Sharpe

(WITT (New Plymouth), g.sharpe@witt.ac.nz)

*The pudding is in the proof: Adult numeracy analysis*

TBA

### Sebastian Schuster

(Victoria University of Wellington, sebastian.schuster@sms.vuw.ac.nz)

*Analytic and Algebraic Analogue Space-Times from Electro-Magnetic Media*

The idea of analogue space-times is to provide an alternative approach to curved space-time physics (and its effects): Instead of observing or venturing to space, one can mimic the relevant behaviour with other physical systems - ideally in a laboratory. Depending on the precise incarnation of the curved space-time one wants to look at, this allows fundamentally different formalisms. Here we shall present two such formalisms arising in the context of electro-magnetic media: On the one hand, one can look for media mimicking the "raising of indices" of a (curved) vacuum metric as encountered in (microscopic) electro-magnetism. This is our example of what we call an "algebraic analogue". On the other hand, one can start with a curved space-time wave equation. Assuming its separability, we can compare one of the resulting ordinary differential equations with a one-dimensional Helmholtz equation with a varying refractive index. This, then, is an "analytic analogue". Analytic and algebraic analogues have different advantages and disadvantages which we shall describe and juxtapose.

### Dominic Searles

(University of Otago, dominic.searles@otago.ac.nz)

*Fundamental slide polynomials and their applications*

We introduce a basis for the polynomial ring called the fundamental slide polynomials. Our aim is to better understand the geometrically-motivated basis of Schubert polynomials, whose structure constants count intersection points of triples of Schubert subvarieties of the complete flag variety. As is the case for the Schubert basis, the structure constants of the fundamental slide basis are nonnegative integers, and we give a nonnegative combinatorial rule for these numbers in terms of shuffles. Moreover, Schubert polynomials expand positively in the fundamental slide basis; we give a combinatorial rule for this expansion. We use these rules to obtain a more compact formula for the product of two Schubert polynomials. We also survey some further applications of and recent developments concerning the fundamental slide basis. (Includes joint work with S. Assaf.)

## Mike Steel

(University of Canterbury, mathmomike@gmail.com)

*Combinatorics of autocatalytic networks*

The notion of a self-sustaining autocatalytic network (called a 'RAF') was originally introduced to investigate simple models for the origin of life. More recently, it has been applied to real (laboratory-based) RNA systems, the metabolic map of bacteria, and to other areas of biology such as ecology. From a combinatorial perspective, a RAF is a subset of vertices (having a special property) within a directed bipartite graph that has two types of vertices and two types of arcs. In this talk, I describe our early and more recent work that led to (i) polynomial-time algorithms to detect RAFs, subRAFs and related structures, and (ii) probabilistic results that describe the precise asymptotic rate of catalysis required for RAFs to arise (joint work with Elchanan Mossel, Wim Hordijk and others).

## Winston Sweatman

(Massey University, Auckland, w.sweatman@massey.ac.nz)

*Mining minerals and cuts of lamb: two recent projects from the Adelaide MISG*

For the last three years, the Australian Mathematics-in-Industry Study Group (MISG) has been held in Adelaide. In 2016 there was a project "Determining a mining sequence to meet blending targets where there is uncertainty in the geological model." In 2018 there was a project "Optimising Carcase cuts in the red meat industry." I will talk about these projects and MISG in general.

## Melissa Tacy

(University of Otago, mtacy@maths.otago.ac.nz)

*Using semiclassical analysis to study eigenfunction problems*

A fundamental problem in the theory of PDE and in harmonic analysis is to study the properties of solutions to

$$Pu = \lambda^2 u$$

where $P$ is a linear differential (or pseudodifferential) operator. In particular we often want to study the growth properties of $u$ as measured, for example, by the $L^p$ norms. Usually this is best understood by decomposing $u$ into contributions from various frequency band. Semiclassical analysis gives us an effective way to study these contributions. In this talk I will discuss the techniques and heuristics of semiclassical analysis and their applications in eigenfunction problems.

## Christopher Tuffley

(Massey University Manawatū, c.tuffley@massey.ac.nz)


*Wada's group-valued link invariants of type 7*

The fundamental group of a knot can be defined in terms of an action of $B_n$, the braid group on $n$ strands, on $F_n$, the free group of rank n. In the early 1990s Wada carried out a computer experiment to look for other actions of $B_n$ on $F_n$ that could be used to define group-valued invariants of knots and links. Up to certain symmetries he found seven types of action, of which four could be used to define link invariants. Of these four, he was left with just one — his group invariants of type 7 — that he was unable to express as the fundamental group of some space associated with the link. We'll explain the above and show that in fact his type 7 group is determined by the branched double cover of the link, answering Wada's question.


## Bruce van Brunt

(Massey University, b.vanbrunt@massey.ac.nz)


*On a growth fragmentation equation with asymmetric division*

Growth fragmentation equations model, among other things, size structured cell populations. Here "size" can be mass, DNA content, or any quantity that is preserved under division. Typically, these models consider only symmetric division, i.e., a cell divides into two daughter cells of equal size. There are, however, cells that divide asymmetrically. In this talk we present a simple model for asymmetric division. Like the symmetric model the underlying differential equation is nonlocal in character. Although much of the analysis for the symmetric case can be adapted for this case there are nonetheless some distinct features particularly when the cell growth is exponential.

**Alna van der Merwe**
(AUT, Alna.vandermerwe@aut.ac.nz)

*A hyperbolic heat conduction model*

The parabolic heat conduction model is considered to be inadequate when modelling heat transfer induced by laser heating. Alternative heat conduction models have been proposed to deal with this situation. The basic assumption of Fourier's law is that the onset of heat flow is instantaneous. The assumption that heat flow starts after a certain time delay or lag time, leads to the Cattaneo-Vernotte model (also called the thermal wave model or the hyperbolic model). However, reliable values for this lag time are not readily available.

A separation of variables approach is an efficient way to investigate the properties of the Cattaneo-Vernotte model. There are several advantages to using this approach rather than using numerical methods. For instance, the associated eigenvalue problem is exactly the same for both models. The convergence of the series solutions is in terms of the so-called energy norm. Partial sums of the series can be used to find approximate solutions for the models, and the accuracy of these approximations depends only on the accuracy of the partial sum approximations of the initial conditions. Consequently, an exact measure of the error at any later time is available.

For some materials it is known that the parabolic heat conduction model provides acceptable results after sufficient time has elapsed. Therefore the hyperbolic model will only be feasible if, after some time, the solution is close to that of the parabolic model. This fact is used to calculate realistic values for the lag time.

The solutions of the two models are compared for the physical problem where a specimen is heated with a short heat pulse applied to one of the boundaries.

**Giorgia Vattiato**
(University of Canterbury, giorgia.vattiato@pg.canterbury.ac.nz)

*Modelling the effect of individual heterogeneity on emergent population characteristics*

There is a growing recognition that, like humans, animals show consistent variation in behaviour among individuals, often described as 'personality'. This individual heterogeneity can lead to significant behavioural differences between members of the same species that can have important consequences for population-level processes and ecological interactions.

However, it is not clear how these behavioural variations contribute to the emergent dynamics of a population and there are critical knowledge gaps around the consequences of this heterogeneity on population responses to disturbance, success of reintroductions, harvest and control, and resource selection.

By using a stochastic, spatially-explicit, individual-based model, I am exploring how consistent inter-individual heterogeneity in behaviour, i.e. different 'personalities', affects the emergent behaviour of a population, my first case-study being the effect of these 'personality' differences on the control of invasive pest-mammals in New Zealand.

The first results of these simulations show a significant difference between density evolution predictions done for a homogeneous population (all individuals are believed to behave the same way) versus a heterogeneous one, indicating that heterogeneity should indeed be taken in consideration when trying to eradicate pests.

This research will benefit experimentalists, conservation managers and businesses developing wildlife management tools by reducing bias and by improving conservation efforts. It will also stimulate integration and cross-fertilisation across a range of scientific and technical disciplines where inter-individual variation influences population dynamics, particularly in human social systems and decision-making.

**Jose Voloch**
(University of Canterbury, felipe.voloch@canterbury.ac.nz)

*Locally recoverable codes and algebraic surfaces*

An error correcting code is a subspace of $k^n$, where $k$ is a finite field. Such a code is said to be locally recoverable with locality $r$ if, for every coordinate of a codeword, its value can be deduced from the value of (certain) $r$ other coordinates. These codes have found many recent applications, e.g., to cloud storage. We will discuss the problem of constructing good locally recoverable codes and present some constructions using algebraic surfaces that sometimes provide codes that are optimal in some precise sense. Joint work with C. Salgado and A. Varilly Alvarado.

**Phillip Wilson**

(University of Canterbury, phillip.wilson@canterbury.ac.nz)

*The story of Maths Craft New Zealand*

Maths Craft aims to bring maths to the masses by celebrating the links between mathematics and craft. Maths Craft New Zealand is a non-profit initiative founded in 2016 and run by University of Canterbury mathematicians Dr Jeanette McLeod and Dr Phil Wilson. In two years we have run nine major events attracting over 10,000 visitors, with exit surveys and other feedback showing increased enthusiasm for mathematics amongst our diverse audience of young and old alike. Noting the growing emphasis on STEAM and the new Digital Curriculum in New Zealand, we have also recently run a pilot programme of professional development to train school teachers to bring maths craft to class. In this talk, we present the story of Maths Craft New Zealand, including our ethos, the practicalities of organising large outreach events, applying for grants, and attracting sponsors. Our aim in the talk is to share what we have learned in the hope that it will inspire and help others, whether they are mathematicians contemplating new outreach initiatives, teachers keen on using maths craft in their classes, or outreach professionals wondering how to put the M in STEAM.

**Wenjun Zhang**

(Auckland University of Technology, wzhang@aut.ac.nz)

*Inferring information from the S&P 500 and CBOE indices*

The Chicago Board Options Exchange (CBOE) updated the CBOE Volatility Index (VIX) in 2003 and further launched the CBOE Skew Index (SKEW) in 2011, in order to measure the 30-day risk-neutral volatility and skewness of the S&P 500 Index (SPX). This paper mainly compares the information extracted from the SPX and CBOE indices in terms of the SPX option pricing performance. Based on our empirical analysis, VIX is a very informative index for option prices. Whether adding the SKEW or the VIX term structure can improve the option pricing performance depends on the model we choose. Roughly speaking, the VIX term structure is informative for some models, while, the SKEW is very noisy and does not contain much important information for option prices.

### 8.3 Posters

**Patrick D. F. Ion (GDML/IMKT & University of Michigan)**
(pion@umich.edu)

*The discrete Fourier transform in (mostly) plane geometry*

There is a relationship between some elementary geometry of the plane and the Discrete Fourier Transform, that can be a starting point for ramifications into the theories of polynomials, circulant matrices and interpolation, and even sculpture and quantum bits. It also leads to new views of the dynamics of point-particle systems in mechanics.

**Valerie Jeong (The University of Auckland)**
(valerie.jeong@auckland.ac.nz)

*Heteroclinic networks with noise and input*

Heteroclinic networks are structures within dynamical systems that have a variety of applications in physical systems, including designing neural networks. The dynamics of heteroclinic networks have been studied in depth for several decades, however, the effects of noise on the networks is not well understood. In my poster, I will present how small noise and/or inputs affect a relatively simple heteroclinic cycle, called the Guckenheimer-Holmes Cycle. This cycle consists of three equilibria and the heteroclinic connections between them. As time evolves, trajectories spend longer and longer time near each equilibrium. When noise or inputs are added, this slowing down no longer occurs. Future work will include studying how the interactions between noise and input affect the cycle, and other dynamical networks.

**Jo Knox (University of Auckland)**
(joknox@hotmail.co.uk)

*Is Mathematical Proof Childproof?*

Mathematical proof is fundamental to the work of mathematicians and yet it is a process that has been marginalised in schools. In this poster, I will outline what might count as proof in the primary classroom and suggest the steps that may be necessary in tasks to support students to engage in proving activity and develop valid reasoning.

**Gray Manicom (University of Auckland)**

(gman522@aucklanduni.ac.nz)

*Noisy Heteroclinic Networks*

Heteroclinic networks are special solutions of dynamical systems in which trajectories cycle between various states, such as saddle type equilibrium solutions or periodic orbits. These heteroclinic networks are structurally stable in cases where the underlying system of ODEs is symmetric, and thus there existing invariant subspaces which contain the connecting heteroclinic orbits. The deterministic behaviour of these systems is, in some cases, well-understood. However, with the addition of noise to the system its behaviour can change significantly. The noisy system may exhibit dynamics such as switching between the network's sub-cycles and a change in the residence times of trajectories near the aforementioned states. This poster will demonstrate some of the effects on the dynamical system which result from the addition of noise to a simple heteroclinic network and pose some of the research questions being investigated by the author.

**Valerie Chopovda (Massey University), Winston Sweatman (Massey University)**

(w.sweatman@massey.ac.nz)

*The family of periodic orbits generated by the 4-body Schubart orbit*

We present a one-parameter planar family of periodic orbits for the equal-mass 4-body problem. This family begins and ends with the Schubart interplay orbit.