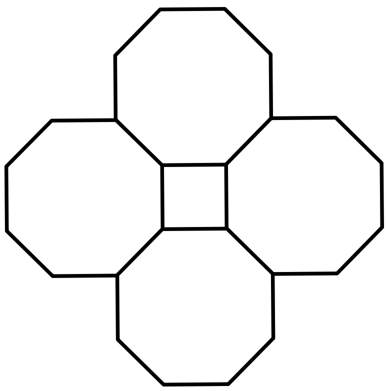
The National Bank

Junior Mathematics Competition

2010 Solutions Booklet



Department of Mathematics and Statistics
University of Otago
Visit us at www.maths.otago.ac.nz/nbjmc

Year 9

First David Wu, Lynfield College
Second David Chou, Macleans College
Third Cherry Yang, St Cuthbert’s College

Top 30

Yi Wang, Auckland Grammar School
Kiran Krishnamurti, Lynfield College
Harrison Fookes, Sacred Heart College (Auckland)
Tim Bullen, Christ’s College
Steven Ling, Lynfield College
Shuo Yang, St Kentigern College
Andrew Bamford, Palmerston North Boys’ High School
Richard Zhang, Scots College
Chris Wang, Hillcrest High School
Stephanie King, St Cuthbert’s College
William Briscoe, Christ’s College
James Chang, Cobham Intermediate School
Michael Davis, Middleton Grange School
Lauren Kelly, Roncalli College
Samuel Attard, Macleans College
Catherine Zheng, St Cuthbert’s College
Simon Ding, Kristin School
Zoe Hu, Macleans College
Cameron Vennell, Napier Boys’ High School
George Dodd, Auckland Grammar School
Nanako Shitara, Columba College
Henry Merton, Northcote College
Logan Rogers-Jenkins, St Kentigern College
Samuel Withers, Christchurch Boys’ High School
Daniel Schipper, Hillcrest High School
Callum Herries, Napier Boys’ High School
Sandra Yuan, Pakuranga College

Year 10

First Byung Cho, Auckland Grammar School
Second Frank Zhou, Macleans College
Third Ian Seong, Burnside High School

Top 30

Hayden White, Shirley Boys’ High School
Nicolaas Waddington, Lynfield College
Evelyn Qian, Diocesan School for Girls
Sheng Cao, Tihoi Venture School
Joon Park, Macleans College
Luke Naylor, Palmerston North Boys’ High School
Sobitha Manoharan, St Cuthbert’s College
Jimmy Yao, King’s College
Jason Cheng, Kristin School
Kevin Yu, Lynfield College
Kevin Choi, Nelson College
Tony Sun, Christ’s College
Jason Fong, Auckland Grammar School
Chang-Chih Jou, ACG Strathallan College
James McElligott, St Peter’s School (Cambridge)
Daniel Britten, St Kentigern College
Joshua Quon, John McGlashan College
Yung Chuen Koh, ACG Strathallan College
Yi Lin Zheng, Diocesan School for Girls
Patrick Gu, St Kentigern College
Fergus O’Leary, King’s High School
Jaehwan Kim, King’s College
Yo-Der Song, Macleans College
Duncan McKee, Scots College
Jonathan Chieng, Auckland Grammar School
Hanzhi Wang, Howick College
Matthew Brown, Whangarei Boys’ High School

Year 11

First	Terence Collins-Hawkins, Samuel Marsden Collegiate School
Second	Arun Chockalingam Shanmuganathan, Auckland International College
Third	Tim Oorschot, Cashmere High School

Top 30

James Allen, Kristin School
Jason Whitworth, ACG Strathallan College
John Kwak, Christ's College
Hannan Kashkari, Lynfield College
Joe Lu, St Kentigern College
Bruce Tsai, Auckland Grammar School
Richard Ngo, King's College
Nalin Choudhary, Lynfield College
Steve Dawson, Napier Boys' High School
Peter Lamborn, Hutt Valley High School
Jacob Swiatek, St Kentigern College
Brydon Sundgren, St Patrick's College (Wellington)
Maggie Pan, Pakuranga College
Robert Shin, Macleans College
Franklin He, Rutherford College
Hyunjin Ahn, Palmerston North Boys' High School
Raymond Lee, Macleans College
Nicholas Twort, St Peter's College (Epsom)
Tushar Taneja, Botany Downs Secondary College
Georgia Nixon, St Cuthbert's College
Andy Chen, Macleans College
Amy Yeh, Pinehurst School
Joey Chen, Rangitoto College
Lucy Fauth, St Kevin's College
Lucy Collingwood, Middleton Grange School
Hun Choi, Rangitoto College
Lea Kapelevich, Epsom Girls' Grammar School

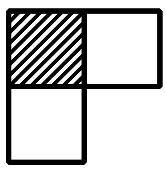
Answers

As usual, we offer one method of answering the questions only. There may be other approaches, and the method shown may not be the most efficient.

The 2010 questions seemed to be the easiest ever. This meant that to score in the top 15%, you virtually had to answer questions 2 and 4 well (these seemed to be fairly easy), and do fairly well in at least one other question, preferably two. Year 9 students (and below) also had to do well in question 1. Several did.

Question One

Rebecca is holding a seminar at the place at which she works. She wants to create an unbroken ring of tables, using a set of identical tables shaped like regular polygons (every side has the same length – such as a square, with four equal sides). Each table must have two sides which completely intersect with the sides of other tables, such as the hatched square table seen to the right. Rebecca plans to put items on display inside the ring where everyone can see them.



(If you cannot name a shape in this question, just give the number of sides. For example, if you think the shape has 235 sides, but don't know the name, just call it a 235-gon - that isn't an answer to any of the parts.)

(a) *Rebecca first decides to use identical square tables. What is the minimum number of square tables placed beside each other so that there is an empty space in the middle?*
Rebecca needs eight tables. This was fairly easy, and done well. A few students forgot that you need a space in the middle, and answered 'nine'.

(b) If Rebecca uses square tables, what shape is left bare in the middle?
A square is left in the middle. Easy, and well answered.

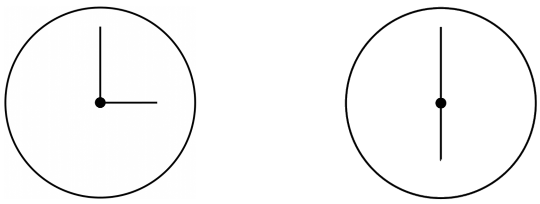
(c) *Rebecca considers using octagon (8 sides) shaped tables.*
(i) What is the minimum number of octagonal tables which Rebecca must have in order for there to be a bare space in the middle so that the tables form an enclosure?
Four octagons. The hardest part of the question, but well answered all the same.

(ii) What is the name given to the bare shape in the middle? If you can't name it, giving the number of sides will be sufficient.
A square is left in the middle. Follow-on was given, but only if a meaningful diagram appeared. We did not give credit for the non-mathematical answer 'diamond'.

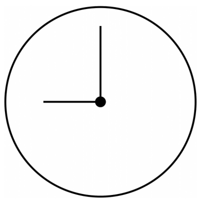
(d) *Apart from squares and octagons, are there any other shaped tables possible? If there are any, name one. If there isn't, say so.*
Several lecturers have looked at this question, and they believe that **any regular polygon** is possible. However, answers like 'rectangle' or 'trapezium' received no credit, as it states at the start that only regular polygons are used.

Question Two

An analogue clock displays the time with the use of two hands. Every hour the minute hand rotates 360 degrees, while the hour hand (which is shorter than the minute hand) rotates 360 degrees over a 12 hour period. Two example times are shown below:



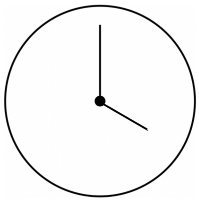
(a) Draw a clock face which shows 9 o'clock. Make sure the hour hand is shorter than the minute hand.



Well answered.

(b) *What is the angle between the two hands at both 3 o'clock and 9 o'clock?*
90°. 270° was accepted, as we did not specify direction. For some reason, many students didn't read the question correctly, and thought we were asking for the angle between the hour hands at 3 o'clock and 9 o'clock. 180° was common (for no marks).

(c) What time to the closest hour (and minute) does the following clock face show?



4 o'clock. Well answered, although a few students mixed their clock hands up, and said 20 past 12.

(d) What is the angle between the two hands at the following times?

(i) *1 o'clock.* **30°.** Most students were able to divide 360° by 12.

(ii) *2 o'clock.* **60°**. Twice the previous answer – follow-on was allowed.

(iii) *Half past one.* **135°**. Several candidates got 150°, not realising that the hour hand moved through 15° in the half hour. No marks for this.

(e) *At what time (to the nearest minute) between 7 and 8 o'clock do the hands meet?* **7:38.**
We cannot believe that this question could be answered with certainty without working. Either an equation had to be set up, or (more commonly) a table showing times either side of the correct answer and the angle between the hands. Half marks for the correct answer with no working.

Question Three

A six digit number "abcdef" *is formed using each of the digits 1, 2, 3, 4, 5, 6 once and only once so that* "abcdef" *is a multiple of 6,* "abcde" *is a multiple of 5,* "abcd" *is a multiple of 4,* "abc" *is a multiple of 3 and* "ab" *is a multiple of 2.*

Note that the correct answers only scored half marks. We had to see some mathematical evidence of how the answers were obtained for the students to get close to full marks. Unfortunately, some of the less experienced markers did 'their own thing', ignoring the mark schedule to give full marks if they saw the answers beyond verification. We have re-marked more than the top 15% of papers, reducing marks where necessary. However many of the lower papers have not been re-marked. The result is 'high' marks for some less capable problem solvers. We apologise for this.

(a) *Find a solution for* "abcdef"*. Show key working.*
Common logic went something like:
e must be 5 (Often seen.)
b, d, f must be even (Also often seen.)
So the only numbers not allocated are 1 and 3. **a and c must be one of these.** (This logic was seldom observed.) Students can now build up a table, *which we must see*, or (less commonly) use logic. If a is set to 1, the answer **123654** emerges.

(b) *Is the solution you found unique (the only possible one)? If it is, briefly explain why. If it isn't, give another solution.*
With a = 3, a second table yields **321654**. (The two answers could be in either order.)
Note that a significant minority of candidates read the question incorrectly and thought that *abcdef* meant a×b×c×d×e×f. They got the answer 720, which certainly does not have six digits. No marks.
123456 was also common, although 1234 is not divisible by 4.

Question Four

A 3 by 2 rectangle is divided up into six equal squares, each containing a bug. When a bell rings, the bugs jump either horizontally or vertically (they cannot jump diagonally and they stay within the rectangle) into a square adjacent to their previous square in any direction, although you cannot know in advance which exact square they will jump into. Every bug changes square; no bug stays put.

As an example, the ordered sextuplet (1, 1, 1, 1, 1, 1) (where this represents the result, not the movement) represents the situation where every bug jumped so that each square still had one bug in it (it could happen). Alternatively, two bugs could also land in the same square. An example (not the only way this could happen) of this might be represented by (2,2,1,0,0,1) – see the diagram to the right. The first number in the sextuplet represents a corner square, the second represents a square on the middle of a side, and so on.

2	2	1
0	0	1

(a) *What is the average number of bugs per square in the 3 by 2 rectangle no matter how the bugs jump?*
6 bags and 6 squares so the average is **1 bug per square**. Generally well answered, although also common were incorrect values. The sort of incorrect 'logic' often seen was this: The boxes must contain 1, 2, or 3 bugs. (For some reason, 0 was ignored,) So the average must be 2.

(b) *From the initial situation of one bug in every square, is it possible for three bugs to end up in the same square if the bell rings only once? If you think it is, write an ordered sextuplet like the two above where this could happen. If you think it can't happen, briefly explain why not.* **Yes.** The most common sextets seen were **(0 3 0 0 3 0)** and **(0,1,0,1,3,1).** But there are several others, marked correct, as well as plenty of incorrect ones. It helped if the number of bugs was six (five was common), or if there weren't three bugs in a corner (also commonly wrong).

(c) *From the initial situation of one bug in every square, it is certainly not possible in a 3 by 2 rectangle for four bugs to end up in the same square if the bell rings only once. Write down the dimensions of the smallest rectangle for which it would be possible.*
A square is a special type of rectangle, despite many students assuring us it's not. So **3×3** is the smallest size. 3×4 or 4×3 was common (for half marks).

(d) *From the initial situation of one bug in every square, five bugs can never end up in the same square if the bell rings only once, no matter the size of the rectangle. In a few words, explain why not.*
Five bugs would have to have five adjacent squares, not counting diagonals – impossible in two dimensions. Well answered. It helped to include a diagram.

(e) *In the 3 by 2 case, how many non-unique sextuplets (like (1, 1, 1, 1, 1, 1)) are possible from the initial situation of one bug in every square, if the bell rings only once? You do not have to list them, although you might like to.*
$2 \times 3 \times 2 \times 2 \times 3 \times 2 = 144$. Not commonly seen. Justification: the bugs in the four corner squares can go 2 ways, and the other two bugs can go three ways, and so $16 \times 9 = 144$.

Question Five

Pania and Rangi exercise weekly by running around two paddocks on their father's farm near Kakanui from A to B to C to D then back to A (see diagram). In a direct line from A to C, the distance is 6250 m. AB is shorter than BC.

This was the 'hard' question at the end, involving 'trig'. A few Year 9s could make progress, but there are also several Year 11s at some schools who don't seem to have met it at all, or so it seemed. No trig was needed in (a) or (c).

(a) *If triangle ABC is a right angled triangle in the ratio of 3:4:5, with B at the right angle, find the lengths of the sides.*

$$\frac{AC}{5} = \frac{AB}{3} = \frac{BC}{4}$$

So $AB = \frac{3}{5}AC$ = **3750 m**

and $BC = \frac{4}{5}AC$ = **5000 m** Not uncommon.

(b) *If triangle ABC is a right angled triangle in the ratio of 3:4:5, with B at the right angle, find the size of angle CAB to one decimal place.*

Angle CAB = $\sin^{-1}(4/5)$

$\approx$ **53.1°** Reasonably common, from Years 10 and 11. The question asked for 1 decimal place, so scale diagrams were not accepted.

(c) *Angle B is in fact a right angle, and AB and BC are whole metres in length, but the sides are not in the ratio of 3:4:5. Find possible lengths for AB and BC.*

Another Pythagorean triple that works is 7-24-25.

This gives AB = (7/25) × 6250 = **1750 m**

and BC = (24/25) × 6250 = **6000 m** Rarely seen. Pythagorean triples are not well known. The worst answered question in the contest, although hardly surprising.

(d) Angle D is not a right angle but is 40°, and CD is 600 m. Use this information to find the length of AD, marked m in the diagram.

Hint: In any triangle XYZ, the following rules apply:

Sine Rule: sin X / x = sin Y / y (= sin Z / z)

Cosine Rule: $x^2 = y^2 + z^2 - 2yz\cos X$

where side x is opposite to angle X, side y is opposite to angle Y, and side z is opposite to angle Z.

One method (there were others) is to use the sine rule twice.

$$\frac{\sin\theta}{600} = \frac{\sin 40^\circ}{6250}$$

$$\theta \approx 3.538^\circ$$

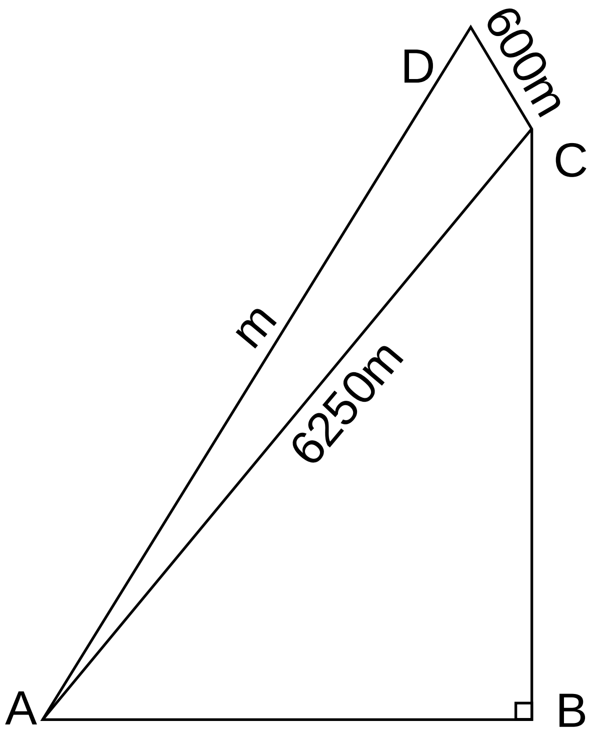
Then the other angle in the triangle is

$180 - \theta - 40 \approx 136.46^\circ$

Apply sine rule again:

$$\frac{\sin 136.46^\circ}{m} = \frac{\sin 40^\circ}{6250}$$

m = **6697.72** (2 d.p.) Uncommon.



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